Contents

1 Introduction

Deadtime effects are barely considered evaluating Mössbauer spectra. Typically the deadtime of the counting system is not known according to which deadtime parameters have been not introduced in programm codes. But as it can be sshown that the baseline of the spectrum hides the deadtime parameter if a detailed knowledge of the geometry of distances of source, diaphragm and detector and its dimensions are taken into account.

2 Baseline

The measurement of the baseline of a Mössbauer spectrum already provides valuable information about the equipment, that is the geometry and dead time effects. In a first step the dependence of the baseline on the position of the moving source shall be considered. Secondly the position of the source dependent on velocity v is described by a parameter *geo* which differ in triangular and sinusoidal mode. The parameter *geo* is well defined by the geometry of the experimental set up. The effectiv value of *geo* from the fit of the spectrum may differ from the geometrical value as a result of dead time effects of the detector and its electronic.

2.1 Solid angle of a circle

The intensity of the radiation from the source S through an aperture proportional to the solid angle Ω determined by the position of the source relative to the aperture. The simplest case is a point source on the axis orthonormal to the aperture as shown in Fig. 1. The surface of the

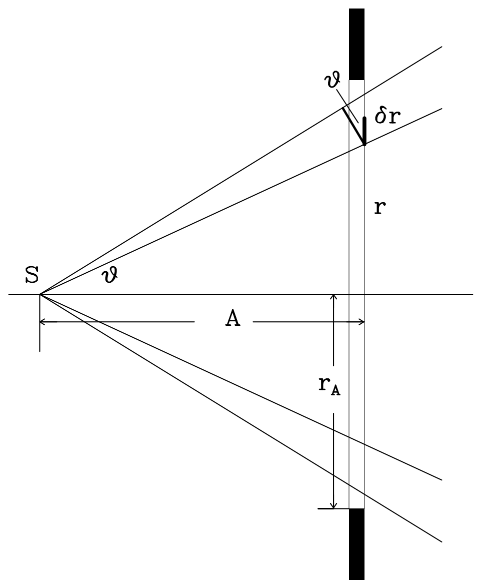

Figure 1: Source at position S, an aperture at a distance A is centered to an optical axis. The solid angle $d\Omega$ belonging to the surface element at r and width δr is equal to the projection of the surface element $2\pi r\delta r \cdot \cos\vartheta$ onto the unit sphere.

infinitesimal ring element from r to $r + \delta r$ of the aperture is $2\pi r\delta r$. The ring element resects from the surface of the sphere with radius $R = \sqrt{r^2 + A^2}$ the element $2\pi r\delta r \cdot \cos\vartheta$. The solid angle $d\Omega$ of the surface element is given by the sphere of radius 1 such that

$$\begin{aligned} d\Omega &= 2\pi r\delta r \cdot \cos\vartheta / R^2 \quad \mathit{with} \\ \cos\vartheta(r) &= \frac{A}{\sqrt{1 + r^2}} \end{aligned}$$

The solid angle is obtained by integration over r.

$$\begin{aligned} \Omega &= 2\pi \int_0^{r_A} \frac{rdr}{(r^2 + A^2)\sqrt{(r^2 + A^2}} \\ &= 2\pi \left(1 - 1/\sqrt{1 + \left(\frac{r_A}{A}\right)^2}\right) \end{aligned}$$

For the experimental cases with ratios $r_A/A << 1$ the Taylor expansion is sufficient. Denoting the area πr_A^2 by F_A the solid angle becomes

$$\Omega = \frac{F_A}{A^2}\left(1 - \frac{3}{4\pi}\frac{F_A}{A^2} + ...\right) \tag{1}$$

The the solid angle changes with the position $A + S(v)$ of the moving source such that the baseline is modulated by $\Omega(S(v))/\Omega(0)$. With $\delta = S(v)/A$ and $\tau = F_A/A^2$ the ratio

$$\frac{\Omega(S)}{\Omega(0)} = \frac{1}{(1+\delta)^2}\left(1 - \frac{57}{16\pi^2}\tau^2 + \frac{3}{2\pi}\tau\delta\right) \tag{2}$$

is well approximated by $1/(1+\delta)^2$ as will be checked for the experiment under consideration.

2.2 The geometry parameter *geo*

In case of a sinusoidal motion of the source $S = S_{max}\sin(\omega t)$ with the distance $r(t) = A + S(t)$ to the aperture the largest distance $A + S_{max}$ is reached at time $t = T/4$ ad at $t = 3/4T$ the distance is $A - S_{max}$. In multiscaling mode the channel number k is proportional to time t such that at the full period time T the counts are collected in the highest channel K. Time t is expressed by channel k according $t = k\cdot T/K$. The velocity $v(t) = dS/dt = S_{max}\omega\cdot\cos(\omega t)$ starts at $t = 0$ with $v_{max} = S_{max}\omega$ and reaches at $t = T/4$ the zero velocity channel.
The geometry effect is well characterized by the parameter $geo = S_{max}/A$.

$$\begin{aligned} \delta(v) &= \frac{S(k)}{A} = \frac{S_{max}}{A}\frac{S(v)}{S_{max}} \\ &= geo \cdot \sin(\omega t) \end{aligned} \tag{3}$$

The sin function is replaced by $\pm\sqrt{1 - cos^2}$ and $\cos(\omega t)$ by $v(t)/v_{max}$ from above to obtain δ as function of v.

$$\begin{aligned} \delta(v) &= \pm geo\sqrt{1 - \left(\frac{v}{v_{max}}\right)^2} \\ geo &= \frac{v_{max}}{2\pi A}T \end{aligned} \tag{4}$$

The last equation uses the identity $\omega = 2\pi T$.
The same consideration for the triangular mode starts at $t = 0$ and $v = v_{max}$. The acceleration b in the opposite direction decreases the velocity to zero in a quarter of a period $T/4$.

$$v(t) = v_{max} + bt$$

Integration of $v(t)$ with the boundary condition $S(t=0)=0$ gives

$$S(t) \;=\; v_{max}t + \frac{1}{2}bt^2 \tag{5}$$

At $t = T/4$ the velocity is zero, so that from Eq. 5 $v_{max} = -bT/4$, and the source gets the maximum deviation S_{max}. Inserting $t = T/4$ and $b = -4v_{max}/T$ the maximal deviation in the triangular mode becomes $S_{max} = v_{max}/8T$. The dependency of S on the velocity v is obtained inserting t from Eq. 5 into $S(t)$ and using $v_{max} = -bT/4$ from above.

$$\begin{aligned} S(v) &= v_{max}\frac{v - v_{max}}{b} + \frac{1}{2}b\left(\frac{v - v_{max}}{b}\right)^2 \\ S(v) &= +S_{max}\left(1 - \left(\frac{v}{v_{max}}\right)^2\right) \end{aligned} \tag{6}$$

The second half period starting at $t = T/2$ with $v(t = T/2) = -v_{max}$

$$v(t) = -v_{max} + b(t - \frac{T}{2})$$

yields the same expression for $S(v)$ with the minus sign. Similar to Eqs.4 for the sinusoidal case the equations for the triangular case are

$$\begin{aligned} \delta(v) &= \pm geo\left(1 - \left(\frac{v}{v_{max}}\right)^2\right) \\ geo &= \frac{v_{max}}{8A}T \end{aligned} \tag{7}$$

Note the differences of the equations Eq.7 and Eq.4.
The approximation of Eq.2 by the prefactor of the bracket

$$\frac{\Omega(S)}{\Omega(0)} = \frac{1}{(1+\delta)^2} \tag{8}$$

can now be jusified. For the aperture of the tube of the conic section of an area $F_A = 28.3mm^2$ at the distances $A = 23mm$ the corrections turns out to be very small compared to 1. With $v_{max} = 5.9\,mm/s$ and $1/T = 4.5Hz$ the ratio $\delta(v = 0) = S_{max}/A$ becomes $7.1 \cdot 10^{-3}$ which together with $\tau = F_A/A^2 = 0.053$ gives $3/2/\pi\delta\tau = 1.8 \cdot 10^{-4}$ and $57/16/\pi^2\tau^2 = 1.0 \cdot 10^{-3}$.

2.3 Dead time effects

Dead time in the detection system reduces the number of counts which are detected. The are two types of detection systems, so called paralyzable and non-paralyzable ones. (also called extendable and non-extendable, resp.). A detailed description can be found in the book of Glenn F. Knoll [1] which is available as a pdf-file. A general test for detection of dead-time distortions in a Poisson-process is described by Jörg W. Müller [2]. Malvin C. Teich [3] considers normalizing transformations for dead-time modified Poisson counting distribution. In case of large dead-times a reductions of the Poisson distribution leading to χ^2-values appreciably below 1. The following considerations are in the limit where between paralyzable and non-paralyzable detection systems cannot yet be differentiated. The non-paralyzable system reduces the count rate N to N_τ by Eq. 9. τ is the dead-time of the system.

$$N_\tau = \frac{N}{1 + N\tau} \tag{9}$$

The paralyzable system reduces the rate exponentially (see Eq. 10). For small $n\tau \leq 0.2$ the expansion of the exponential is still a good approximation and the system behaves like a non-paralyzable one.

$$\begin{aligned} N_\tau &= Ne^{-N\tau} \\ &= N/e^{N\tau} \\ &\approx \frac{N}{1 + N\tau} \end{aligned} \tag{10}$$

The approximation of non-paralyzable counters is assumed in the following discussion of the shape of the baseline. The reversed relation

$$N = \frac{N_\tau}{1 - N_\tau \tau} \tag{11}$$

is of later use.

The number of counts/sec at the detector is modulated by the ratio of solid angles from source position $S(v)$ at velocity v and at $S(v = \pm v_{max}) = 0$.

$$N(S(v)) = N(S = 0)\frac{\Omega(S(v))}{\Omega(S = 0)}$$

and according to Eq. 8

$$N(v) = \frac{N(S=0)}{(1+\delta(v))^2} \qquad (12)$$

Dead time effects reduce the rate $N(v)$ which is counted by the detector (Eq. 9) to

$$N_\tau(v) = \frac{N(v)}{1+N(v)\tau} \qquad (13)$$

Inserting Eq.12 with $N_0 = N(S=0)$ gives

$$\begin{aligned} N_\tau(v) &= \frac{N_0}{(1+\delta(v))^2} / \left(1 + \frac{N_0\tau}{(1+\delta(v))^2}\right) \\ &= \frac{N_0}{(1+\delta(v))^2 + N_0\tau} \end{aligned} \qquad (14)$$

Three parameter N_0, τ and *geo* of the function $\delta(v)$ determine the baseline. The count rate N_0 shall be replaced by the detected rate, such that the number of counts $N_\tau(0)$ at $S = 0$ becomes a fit parameter. $N_{\tau 0} = N_\tau(0)$ from Eq.13 by use of the reversed relation of Eq.11 inserted in Eq.14 gives

$$\begin{aligned} N_\tau(v) &= \frac{N_{\tau 0}}{1-N_{\tau 0}\tau} / \left((1+\delta(v))^2 + \frac{N_{\tau 0}\tau}{1-N_{\tau 0}\tau}\right) \\ &= \frac{N_{\tau 0}}{(1+\delta(v))^2 - N_{\tau 0}\tau((1+\delta(v))^2 - 1)} \end{aligned} \qquad (15)$$

$N_\tau(v)$ can be well approximated by two parameters $N_{\tau 0}$ and effective geometry parameter geo_{eff} such that there is no deadtime parameter as Eq.12

$$N_\tau(v) = \frac{N_{\tau 0}}{(1+\delta_{eff}(v))^2} \qquad (16)$$

Equating $N_\tau(v)$ of Eq.16 and Eq.15 and considering only linear terms of δ and δ_{eff} the relation

$$\delta_{eff}(v) = \delta(v) \cdot (1 - N_{\tau 0}\tau)$$

is obtained which lead to similar relationship for the geo parameters by Eqs. 4 and 7

$$geo_{eff} = geo \cdot (1 - N_{\tau 0}\tau) \qquad (17)$$

The effectiv value is obtained from the spectrum without taking deadtime into account. The *geo*-parameter uncovers the deadtime of the measurement. Note that independent of the deadtime of the detector $1 - N_{\tau 0}\tau$ is never less than 0 since according to Eq. 11 this value is $1/(1 + N_0\tau)$.

Using parameter *geo* not as fit parameter but as known from Eq.7 the product of the count rate at $S = 0$ and the deadtime $N_{\tau 0}\tau$ is evaluated already from the shape of the baseline. The fact that the intensities of the absorption lines of the spectrum are affected by the deadtime as is the baseline impresses the neccessity for deadtime corrections which are easily accessible by the *geo*-parameter.

3 The product $N\tau$

The product $N\tau$ as evaluated by the shape of the baseline of the Mössbauer spectrum or by fits of the measured count rate versus the true count rate do not depend on the energy window for the selected counts. Although the energy window around the Mössbauer transition, 14.4keV in case of $^{57}Co(Rh)$-source, is choosen as narrow as possible in order to minimize non resonant γ-quanta contributing to the background the relative loss of true counts is the same as counting the full energy spectrum. Fig.2 shows the γ-spectrum of a $^{57}Co(Rh)$-source obtained with a sodium iodide (NaI) scintillation counter. The low count rate at large distance

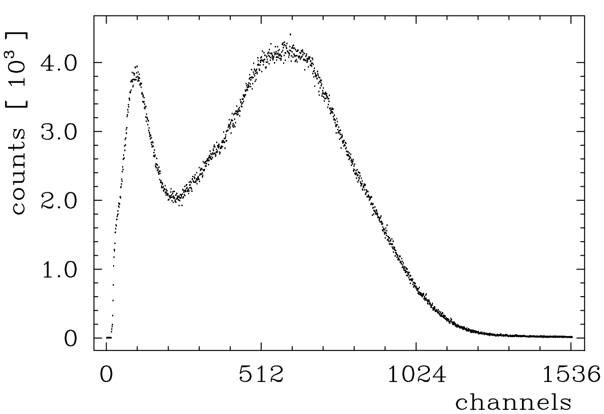

Figure 2: γ-spectrum of a $^{57}Co(Rh)$-source obtained with a sodium iodide (NaI) scintillation counter. The narrow peak is the 14.4keV energy of the Mössbauer radiation. The broad line stems from 122keV and 136keV and the background of their Compton radiation inside the NaI-crystal.

was defined as the true rate without deadtime loss. At half the distance the expected increase by a factor 4 (according to $1/r^2$) justified this assumption. Fig.3 demonstrates deadtime count loss for the full energy spectrum (ungated) and for the gated spectrum of the Mössbauer window. The increase of the detected counts is plotted versus the linear scale of the solid angle $\Omega/4\pi$ corresponding to the decreasing distances shown on top for each measurement. The straigt line represents no count loss which is the same by the different scales on the left for the ungated counts and the gated ones on the right. Within the error the detected counts are on the same curve which is fitted with the product $N\tau = 0.22$.

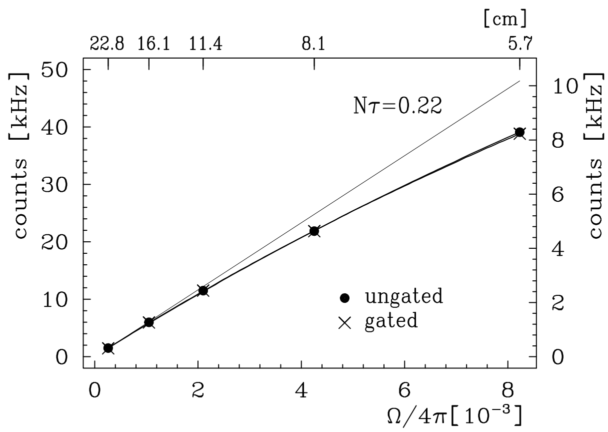

Figure 3: The count loss by the deadtime of the scintillator counting system (see Fig.2) for two energy windows, the full energy spectrum (ungated) and for the gated spectrum of the Mössbauer window. The straigt line means no count loss corresponding to Deadtime $\tau = 0$. The value $N\tau$ reaches 0.22 at the rate measured at the solid angle $\Omega/4\pi = 8 \cdot 10^{-3}$ which is about $50kHz$ for the ungated case (scale on the left ordinate) and $8kHz$ for the Mössbauer window (scale on the right one). The two curves match with in the error.

The value of $N\tau$ provides an answer for the nonparalyzable deadtime τ of the detection system in the limit of large rate N

$$N_\tau = \frac{N}{(1 + N\tau)} \propto \frac{1}{\tau} \tag{18}$$

The detected rate for paralyzable deadtime behaviour reaches a maximum at $1/\tau$.

4 Hybrid model of paralyzable and non-paralyzable deadtime

The non-paralyzable deadtime could be easily handled by the fact that the detected rate can be expressed by the true rate and vice versa (Eqs. 9,11). Using the hybrid model [4]

$$N_\tau = N e^{-N\tau_P} \frac{1}{1 + N\tau_N} \tag{19}$$

with two deadtimes τ_P and τ_N the rate $N_\tau(v)$ at velocity v cannot be expressed anymore by the rate $N_{\tau 0}$ at the position $S = 0$ of the source. Inserting the rate $N(v)$ from the source of Eq. 12

$$\begin{aligned} N_\tau(v) &= \frac{N_0}{(1+\delta)^2} exp\left(-\frac{N_0\tau_P}{(1+\delta)^2}\right) \frac{1}{1 + \frac{N_0\tau_N}{(1+\delta)^2}} \\ &= \frac{N_0}{(1+\delta)^2 + N_0\tau_N} exp\left(-\frac{N_0\tau_P}{(1+\delta)^2}\right) \end{aligned} \tag{20}$$

$N_\tau(v)$ depends on 3 parameters, N_0 and 2 deadtimes, each one changes the value $N_{\tau 0}$ which is directly determined by the baseline at $S = 0$.

$$N_{\tau 0} = N_0 e^{-N_0\tau_P} \frac{1}{1 + N_0\tau_N} \tag{21}$$

Taking $N_{\tau 0}$ and the products $N_0\tau_P$,$N_0\tau_N$ as parameters the following equation replaces Eq. 15

$$N_\tau(v) = N_{\tau 0} \frac{1 + N_0\tau_N}{(1+\delta)^2 + N_0\tau_N} exp\left(-\frac{N_0\tau_P}{(1+\delta)^2}(1 - (1+\delta)^2)\right) \tag{22}$$

5 Mössbauer spectrum

5.1 Absorption

So far the dependence of the baseline on the solid angle by $\delta(v)$ and the deadtimes τ_P and τ_N are considered. The absorber decreases by its Mössbauer spectrum the count rate at various velocities v. The rate $N_0 = N(v = v_{max})$ in Eq. 15 becomes $N_0 \cdot M(v)$ where

$$\begin{aligned} M(v) &= 1 - f_r(1 - T(v)), \quad T(\infty) = 1 \\ f_r &= f \cdot (1 - b_f) \end{aligned} \tag{23}$$

The fraction of resonant counts f_r is a product of the Lamb-Mössbauer factor f of the source and the fraction of Mössbauer transitions $(1 - b_f)$, such that b_f is the fraction of background counts. The Mössbauer spectrum is obtained replacing N_0 by the product $N_0 M(v)$ in Eq. 20

$$N_\tau(v) = \frac{N_0 M(v)}{(1+\delta)^2 + N_0 M(v)\tau_N} exp\left(-\frac{N_0 M(v)\tau_P}{(1+\delta)^2}\right) \tag{24}$$

The value N_0 is defined by Eq. 21, the rate at v_{max} where $M(v_{max}) = 1$.

$$\begin{aligned} N_\tau(v) &= N_{\tau 0} M(v) \frac{1 + N_0\tau_N}{(1+\delta(v))^2 + M(v) N_0 \tau_N} \\ &\quad \cdot exp\left(-N_0\tau_P(M(v)/(1+\delta(v))^2 - 1)\right) \end{aligned} \tag{25}$$

$N_{\tau 0}$ from Eq. 21 is not affected by the Mössbauer spectrum if v_{max} at $S = 0$ is large enough where the baseline is reached. The parameters $N_0\tau_N$ and $N_0\tau_P$ are determined by the baseline. The function $\delta(v)$ is known from the geometry, the drive mode and freqeuncy. The Mössbauer spectrum obviously can be strongly affected by the deadtime parameters.

5.2 emission

The scatterer inceases by its Mössbauer spectrum the count rate at various velocities v. The rate is the sum of the rate of the baseline plus the counts $N_{rc}M(v)$ generated by resonance scattering

$$N(v) = N_b + N_{rc} M(v) \qquad \sum_k M(v(k)) = 1$$

$M(v = \infty) = 0$ and N_{rc} are the resonant generated counts. The number of detected counts with deadtime τ and v-dependence by the geometry effect

$$\begin{aligned} N_\tau(v) &= \frac{N_b + N_{rc}M(v)}{(1+\delta)^2 + (N_b + N_{rc}M(v))\tau} \\ N_\tau(\infty) &= \frac{N_b}{1 + N_b\tau} \end{aligned}$$

$N_\tau(\infty)$ is the measured rate of the baseline $N_{\tau b}$ which shall be a fit parameter and also instead of τ again a product $N_b\tau$. Replacing N_b gives

$$N_\tau(v) = \frac{N_{\tau b}(1 + N_b\tau) + N_{rc}M(v)}{(1+\delta)^2 + (N_{\tau b}(1 + N_b\tau) + N_{rc}M(v))\tau}$$

Replacing further N_{rc} by $N_{\tau rc}/(1-N_{\tau rc}\tau)$ and the deadtime τ by

$$\begin{aligned}\tau &= \frac{N_{\tau b}\tau}{N_{\tau b}} \qquad N_{\tau b}\tau = \frac{N_b\tau}{1+N_b\tau}\\ &= \frac{1}{N_{\tau b}}\frac{N_b\tau}{1+N_b\tau}\end{aligned}$$

$N_\tau(v)$ depends on the parameters $N_{\tau b}$, $N_{\tau rc}$ directly obtained from the spectrum and $\Theta = N_b\tau$ proportional to the deadtime.

$$N_\tau(v) = N_{\tau b}\frac{(1+\Theta)(1+N_{\tau rc}M(v)/((N_{\tau b}-N_{\tau rc})\Theta+N_{\tau b}))}{(1+\delta)^2+(1+N_{\tau rc}M(v)/((N_{\tau b}-N_{\tau rc})\Theta+N_{\tau b})\Theta} \tag{26}$$

6 Appendix

6.1 Derivatives transmission

Analytical derivatives speed up the search for the minimum of χ^2 of fit routines as compared to numerical ones. For several parameters of Eq. 25 the derivatives are readily obtaind. Some abbreviations an relations derived from Eq. 23 are collected:

$$\begin{aligned}\Theta_N &= N_0\tau_N\\ \Theta_P &= N_0\tau_P\\ g_f &= (1+\delta(v))^2\\ \frac{\partial M(v)}{\partial b_f} &= \frac{1-M(v)}{1-b_f}\\ \frac{\partial M(v)}{\partial f} &= -\frac{1-M(v)}{f}\end{aligned}$$

Eq. 25 takes the shape

$$N_\tau(v) = N_{\tau 0}M(v)\frac{1+\Theta_N}{g_f+M(v)\Theta_N}\cdot exp\left(-\Theta_P(1-M(v)/g_f)\right)$$

The following 7 derivatives are used in the code of the program.

$$\begin{aligned}\frac{\partial N_\tau(v)}{\partial N_{\tau 0}} &= \frac{N_\tau(v)}{N_{\tau 0}}\\ \frac{\partial N_\tau(v)}{\partial \Theta_N} &= N_\tau(v)\left(\frac{1}{1+\Theta_N}-\frac{M(v)}{g_f+M(v)\Theta_N}\right)\\ \frac{\partial N_\tau(v)}{\partial \Theta_P} &= -N_\tau(v)(1-M(v)/g_f)\end{aligned}$$

$$
\begin{aligned}
\frac{\partial N_\tau(v)}{\partial g} &= -N_\tau(v)\left(\frac{1}{g_f + M(v)\Theta_N} - \frac{M(v)\Theta_P}{g_f^2}\right)\cdot 2(1+\delta(v))\frac{\partial\delta(v)}{\partial g} \\
\frac{\partial N_\tau(v)}{\partial b_f} &= N_\tau(v)\frac{1-M(v)}{1-b_f}\left(\frac{1}{M(v)} - \frac{1}{g_f + M(v)\Theta_N} + \frac{\Theta_P}{g_f}\right) \\
\frac{\partial N_\tau(v)}{\partial f} &= -N_\tau(v)\frac{1-M(v)}{f}\left(\frac{1}{M(v)} - \frac{1}{g_f + M(v)\Theta_N} + \frac{\Theta_P}{g_f}\right)
\end{aligned}
$$

The value of $g_f = (1+\delta(v))^2$ and the derivative $\partial\delta(v)/\partial g$ multiplied by $2(1+\delta(v))$ are provided by the "subroutine drive()".

6.2 Derivatives emission

The complicated structure of E. 26 leads to clumsy partial analytical derivatives, which are not any more advantages over numerical ones. The partial derivative of $\partial/\partial g$ for the geometry parameter is an exception as the derivative

$$
\frac{(1+\delta(v))^2}{\partial g} = 2(1+\delta(v))\frac{\delta(v)}{\partial g} \tag{27}
$$

is provided by the 'subroutine drive()'.

$$
\frac{\partial N_\tau(v)}{\partial g} = -\frac{N_\tau(v)}{d_N}2(1+\delta(v))\frac{\delta(v)}{\partial g} \tag{28}
$$

$$
d_N = (1+\delta)^2 + (1 + N_{\tau rc}M(v)/((N_{\tau b} - N_{\tau rc})\Theta + N_{\tau b})\Theta \tag{29}
$$

d_N denotes the denomenator in Eq. 26.

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