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Indirect couplings between magnetic layers in a metallic half-space

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Abstract

For an ideal and degenerate electron gas confined to a half-space, indirect couplings between two parallel ferromagnetic layers or slabs are derived. © 1999 Elsevier Science B.V. All rights reserved.

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1. The homogeneous case

This paper derives the indirect interaction between ferromagnetic films in a half-space. To this end we recall the formulas which apply to the homogeneous metal. Consider a spin-dependent point interaction

$$H_{\text{point}} = -\gamma\delta^3(\boldsymbol{r}-\boldsymbol{a})\boldsymbol{I}\cdot\frac{\sigma}{2}, \tag{1}$$

where the coupling constant γ has the dimension energy × volume, and where $\boldsymbol{r}$ is the position and $\sigma/2$ the dimensionless spin operators of an electron, while $\boldsymbol{I}$ is the direction of an ion spin at position $\boldsymbol{a}$. Eq. (1) is often used to represent the exchange interaction of an ion spin with conduction electrons. When a homogeneous, degenerate and ideal electron gas is acted upon by H_{point}, a spin density (angular momentum per unit volume) [1] known as Ruderman–Kittel–Kasuya–Yosida (RKKY) polarization $\boldsymbol{P}(\boldsymbol{x}) = \boldsymbol{I}AR(u)$ appears:

$$A = \frac{\gamma k_F^4 m}{2\pi^3\hbar^2}, \quad R(u) = \frac{\sin(u) - u\cos(u)}{u^4}. \tag{2}$$

Here $u = 2k_F|\boldsymbol{x}-\boldsymbol{a}|$ is the distance to the position $\boldsymbol{a}$ measured in units $1/(2k_F)$, where k_F is the Fermi wave number of the degenerate electron gas. When another ion spin is at position $\boldsymbol{b}$ with coupling constant γ' and spin direction $\boldsymbol{I}'$, it interacts with the polarization at $\boldsymbol{b}$ and is therefore indirectly coupled to the ion at $\boldsymbol{a}$ with *ion–ion coupling energy*

$$-\gamma' A\boldsymbol{I}\cdot\boldsymbol{I}'R(2k_F|\boldsymbol{b}-\boldsymbol{a}|). \tag{3}$$

This procedure is valid in lowest-order perturbation theory, which is a very good approximation in metals where $\gamma k_F^3 < \hbar^2 k_F^2/(2m)$.

As a next step, we consider the indirect interaction between parallel films in an infinite metal. The action of a ferromagnetic plane with x-coordinate a and spin direction $\boldsymbol{I}$ on the conduction electrons has the form

$$H_{\text{plane}} = -\beta\delta(x-a)\boldsymbol{I}\cdot\frac{\sigma}{2}, \tag{4}$$

where the coupling constant β has dimension energy × length. If there are ν point couplings, Eq. (1), per unit surface, then $\beta = \nu\gamma$. The resulting interaction energy per unit surface between two *parallel planes* separated by a distance $|a-b|$ is obtained by integrating $R(|\boldsymbol{a}-\boldsymbol{b}|)$ with fixed $\boldsymbol{a}$ over all positions $\boldsymbol{b}$ in the second film [2,3]:

$$\eta = -\boldsymbol{I}\cdot\boldsymbol{I}'\nu'\gamma'\nu\, A2\pi\int_0^\infty \mathrm{d}s\, s\, R(2k_F\sqrt{s^2+(b-a)^2}) \tag{5}$$

$$= \Gamma\int_{2k_F|b-a|}^\infty \mathrm{d}z\, z\, R(z) \tag{6}$$

$$= \Gamma\, G(2k_F|a-b|) \tag{7}$$

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with

$$\Gamma = -\beta\beta' \boldsymbol{I}\cdot\boldsymbol{I}' k_F^2 m/(2\pi\hbar)^2, \tag{8}$$

$$G(v) = \frac{\pi}{4} - \frac{Si(v)}{2} + \frac{\sin(v) - v\cos(v)}{2v^2}. \tag{9}$$

This is the minimum work required to bring the ferromagnetic magnetizations into the configuration described by $\boldsymbol{I}$ and $\boldsymbol{I}'$ with constant Fermi energy. The integration describes ion spins which are homogeneously distributed in the plane. The actual positions of finite ions may lead to deviations.

In this linear theory, the interaction of a *plane* at $x = a$ *with a ferromagnetic half-space* for x values from $b(> a)$ to ∞ is obtained by an integration of $G(v)$ for v from $2k_Fb$ to ∞:

$$\eta_1 = \Gamma_{1a} G_1[2k_F(b-a)] \tag{10}$$

$$\Gamma_{1a} = -\beta\alpha' \boldsymbol{I}\cdot\boldsymbol{I}' k_F^2 m/(2\pi\hbar)^2, \tag{11}$$

$$G_1(v) = -\left(\frac{\pi}{4} - \frac{Si(v)}{2}\right)v + \frac{\sin(v) + v\cos(v)}{2v}. \tag{12}$$

Here

$$\alpha' = \gamma'\mu'/(2k_F), \tag{13}$$

where μ' is the number of point couplings γ' per unit volume. α and β have the same dimension: energy × length. The label of Γ_{1a} indicates that the first ferromagnet is a plane.

If the second ferromagnet is a *slab* of finite thickness, $b_1 < x < b_2$, the interaction energy with the *plane* at a with $a < b_1 < b_2$ (or $b2 < b1 < a$) is

$$\eta_1(2k_F|a - b_1|) - \eta_1(2k_F|a - b_2|). \tag{14}$$

A further integration of Eq. (12) gives the coupling between *two ferromagnetic half-spaces* separated by a spacer of thickness $|b - a|$:

$$\eta_2 = \Gamma_2 G_2(2k_F|b-a|), \tag{15}$$

$$\Gamma_2 = -\alpha\alpha' \boldsymbol{I}\cdot\boldsymbol{I}' k_F^2 m/(2\pi\hbar)^2, \tag{16}$$

$$G_2(v) = \left(\frac{\pi}{4} - \frac{Si(v)}{2}\right)\left(1 + \frac{v^2}{2}\right) - \frac{\sin(v) + v\cos(v)}{4}. \tag{17}$$

The functions are related by

$$-\frac{d}{dv}G_2(v) = G_1(v), \quad -\frac{d}{dv}G_1(v) = G(v). \tag{18}$$

They are finite at the origin:

$$G(0) = \frac{\pi}{4}, \quad G_1(0) = 1, \quad G_2(0) = \frac{\pi}{4}. \tag{19}$$

Fig. 1 shows their space dependence.

A ferromagnetic *slab* with x values in the interval $a_1 < x < a_2$ interacts with a *half-space*, $x > b$, where $b > a_2$, with the energy

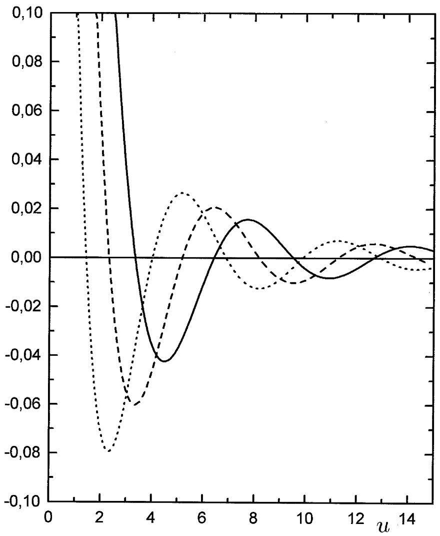

Fig. 1. The dimensionless functions, which describe the dependence of the interactions of two ferromagnets in an infinite metal on the spacer thickness: (full line) $G(u)$, for two planes, (dashed line) $G_1(u)$, for a plane and semi-space, (dotted line) $G_2(u)$, for two semi-spaces.

$$\eta_2[2k_F(b - a_2)] - \eta_2[2k_F(b - a_1)]. \tag{20}$$

If there are *two slabs* for x values $a_1 < a_2$ and $b_1 < b_2$, their interaction is given by

$$\eta_2(2k_F|b_1 - a_2|) - \eta_2(2k_F|b_1 - a_1|) - \eta_2(2k_F|b_2 - a_2|) + \eta_2(2k_F|b_2 - a_1|). \tag{21}$$

2. The half-space

The spin polarization $P_h(x, y, z)$ of an electron gas in a half-space, $x > 0$, induced by a point coupling, Eq. (1), has been calculated in Ref. [4]. It can be expressed with Eq. (2) using the distances ρ_- to the acting point $\{a, a_y, a_z\}$, and ρ_+ to a point in the mirror position $\{-a, a_y, a_z\}$:

$$\rho_\mp = \sqrt{(x \mp a)^2 + (y - a_y)^2 + (z - a_z)^2}. \tag{22}$$

In the half-space, $R(u)$ in Eq. (2) is replaced by

$$R(2k_F\rho_-) + R(2k_F\rho_+) - \frac{(\rho_- + \rho_+)^2}{2\rho_-\rho_+} R[k_F(\rho_- + \rho_+)]. \tag{23}$$

To obtain the interaction between two ferromagnetic planes with x-coordinates a and b, respectively, the expression, Eq. (23), is to be integrated over the variables

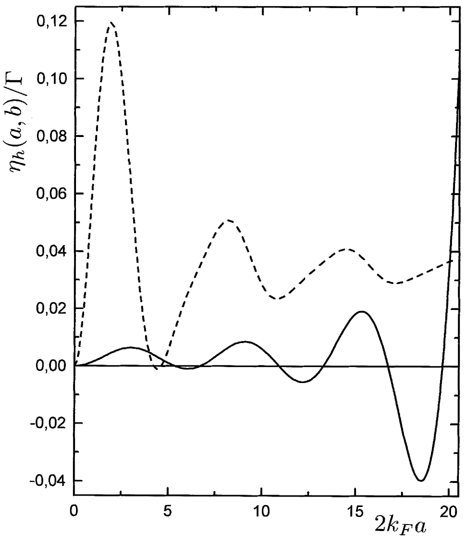

Fig. 2. Dimensionless coupling $\eta_h(a, b)/\Gamma$ of two magnetic planes in a metallic half space $x > 0$: (full line) for a fixed position $2k_F b = 23$ of one plane as a function of the position $2k_F a$ of the other plane, (dashed line) for a fixed spacing $2k_F(b - a) = 3$ between the planes as a function of the thickness $2k_F a$ of the covering spacer. The second term of Eq. (23) then oscillates with period π. In this graph this only deforms the usual oscillations with period 2π.

y and z with $x = b$. For the first two terms in Eq. (23) this is exactly the passage from Eq. (2) to Eq. (9). This is also true for the last term, since $w = \rho_- + \rho_+$ varies with $t = (y - a_y)^2 + (z - a_z)^2$ as $\partial w/\partial t = (\rho_- + \rho_+)/(2\rho_-\rho_+)$. So the two planes couple as [5].

$$\eta_h = \Gamma\{G(2k_F|a - b|) + G[2k_F(a + b)] - 2G[k_F(|a - b| + a + b)]\}. \qquad (24)$$

Thus, the coupling between two ferromagnetic layers in a half-space can be expressed by that in free space: it is as if the plane which is more distant to the boundary were coupled to the other plane in its normal position, plus to that plane in its mirror position, minus twice to that plane at the boundary of the half space. Fig. 2 shows this interaction for a fixed position b as a function of a for $0 < a < b$, where the surface of the half-space is at $x = 0$, and also for a fixed separation $b - a$ of the ferromagnetic planes as a function of the thickness a of the spacer cover.

As in the homogeneous case, if one or both of the ferromagnets have finite thickness or even fill a half-space, the resulting interactions can be obtained by integrations over a or/and b. Since the way how a and b enter the three terms of η_h depends on which is larger, we shall in the following assume:

$$0 < a < b. \qquad (25)$$

The coupling of a ferromagnetic *plane* at a with a ferromagnetic *half-space*, $x > b$, becomes with Eq. (12)

$$\eta_{h1}(a, b) = \Gamma_{1a}\{G_1[2k_F(b - a)] + G_1[2k_F(b + a)] - 2G_1(2k_F b)\}. \qquad (26)$$

Therefore, the interaction of ferromagnetic *plane* at $x = a$ with a *slab* in the interval $b_1 < x < b_2$ becomes

$$\eta_{h1}(a, b_1) - \eta_{h1}(a, b_2). \qquad (27)$$

Integrating Eq. (24) over a gives the coupling of a *slab* in the interval $a_1 < x < a_2$ with a *plane* at b:

$$\Gamma_{1b}\{G_1[2k_F(b - a_2)] - G_1[2k_F(b - a_1)] + G_1[2k_F(b + a_2)] - G_1[2k_F(b + a_1)] - 4k_F(a_2 - a_1)G(2k_F b)\}. \qquad (28)$$

$$\Gamma_{1b} = -\alpha\beta' \boldsymbol{I}\cdot\boldsymbol{I}' k_F^2 m/(2\pi\hbar)^2. \qquad (29)$$

The index of Γ_{1b} indicates that the second ferromagnet is a plane. The last term of Eq. (28) contains the function defined in Eq. (9). Eq. (28) is quite different from Eq. (27), since with $b > a$ the last term in Eq. (24) depends on b only.

Two parallel slabs with $0 < a_1 < a_2 < b_1 < b_2$ interact with the energy

$$\Gamma_2\{G_2[2k_F(b_2 - a_2)] - G_2[2k_F(b_1 - a_2)] - G_2[2k_F(b_2 - a_1)] + G_2[2k_F(b_1 - a_1)] + G_2[2k_F(b_2 + a_2)] - G_2[2k_F(b_1 + a_2)] - G_2[2k_F(b_2 + a_1)] + G_2[2k_F(b_1 + a_1)] - 4k_F(a_2 - a_1)[G_1(2k_F b_2) - G_1(2k_F b_1)]\}. \qquad (30)$$

A last constellation consists of a ferromagnetic *slab* at $a_1 < x < a_2$ interacting with a *half-space* $x > b_1$. This is the previous case, Eq. (30), for $b_2 = \infty$, i.e.,

$$\Gamma_2\{-G_2[2k_F(b_1 - a_2) + G_2[2k_F(b_1 - a_1)] - G_2[2k_F(b_1 + a_2)] + G_2[2k_F(b_1 + a_1)] + 4k_F(a_2 - a_1)G_1(2k_F b_1)\}. \qquad (31)$$

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