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Properties of synchrotron radiation

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This article describes the basic mechanisms for the generation of synchrotron radiation. It gives the basic equations for the emission characteristics of a single transversally accelerated relativistic particle as well as the modifications to these equations due to the multi-particle behaviour of real accelerator beams. It also introduces the boundary conditions for emission of coherent radiation and at the end gives an overview of the parameters of synchrotron radiation sources presently used for nuclear resonance scattering.

1. Introduction

Synchrotron radiation was first observed in the General Electric 70 MeV synchrotron in 1947. Since then the name synchrotron radiation is in use despite the fact that nowadays the term synchrotron radiation is generalized to all electromagnetic radiation generated by transverse acceleration of relativistic charged particles.

Transverse acceleration can be produced by all types of forces acting on charged particles but for practical purposes almost exclusively static transverse magnetic fields are employed. Furthermore, as charges only electrons or positrons will be considered. Since for a particle of charge *e* and of momentum $\vec{p}$, with $\vec{v}$ its velocity, the acceleration by a transverse magnetic field $\vec{B}$ is given by the Lorentz force $\vec{F}_L$:

$$\vec{F}_L = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = e \cdot \left[\vec{v} \times \vec{B}\right], \tag{1.1}$$

and because the acceleration $\mathrm{d}\vec{p}/\mathrm{d}t$ fully determines all characteristics of the emitted radiation, the synchrotron radiation can be purpose-engineered by selecting the type of particle, the particle current $I(t)$, the particle energy E and the spatial configuration of the magnetic fields. Since all these engineering parameters can be varied over a very wide range the resulting synchrotron radiation can be tailored in frequency, polarization, time structure, source sizes and emission angles to be the ideal probe to a large variety of experimental problems in physics, biology and applied sciences. This possibility to tailor-engineer the probing radiation is at the origin of the world wide increase of usage of synchrotron radiation in ever increasing areas of research in the last few decades.

The basic theory of synchrotron radiation has been known for almost 100 years as described in the original papers of Liénard [1] and Schott [2]. Modern comprehensive presentations of the theory of synchrotron radiation can be found in several textbooks (e.g., Wiedemann [3]) and need therefore not be repeated here. Instead a short, qualitative and more intuitive introduction to the physical mechanism of synchrotron radiation is given in this section, while quantitative information for users of synchrotron radiation together with the technical arrangements and constraints will be given in the following sections.

2. Basic properties

Synchrotron radiation – like any other electromagnetic radiation – is generated by accelerating electrically charged particles. Let us start with the emission of radiofrequency waves from a dipole antenna: here the emitting particles with charge e are quasifree electrons of the Fermi sea of the metal structure, which are accelerated along the direction of the antenna under the electric forces created by the transmitter. As long as the antenna is at rest in the frame of the observer the radiated power P in a solid angle $\mathrm{d}\Omega = \mathrm{d}\Theta\,\mathrm{d}\phi$ is given by classical nonrelativistic electrodynamics:

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{e^2}{4\pi c^3} \cdot \dot{v}^2 \cdot \sin^2\Theta \qquad (2.1)$$

and the total radiated power is given by

$$P = \frac{2}{3}\frac{e^2\dot{v}^2}{c^3} \quad \text{(Larmor's formula)}. \qquad (2.2)$$

The emission is characterized by:

- maximum power radiated perpendicular to the direction of the acceleration,
- zero radiation in the direction of acceleration,
- polarization along the direction of acceleration.

The emitted frequency spectrum depends on the time dependence of the acceleration experienced by the observer. If the observer sees a continuous harmonic acceleration of the charge with one frequency f_0 also the emitted electromagnetic radiation has the same frequency f_0. If the observer would see the same source through a mechanical shutter, he would observe additional frequencies corresponding to the modulation characteristics and if the antenna would be powered only by a single short pulse of duration δt the observer would see a frequency spectrum according to the Fourier decomposition of the pulse with a maximum frequency

$$f_{\mathrm{max}} = 1/\delta t. \qquad (2.3)$$

The term synchrotron radiation is used when the antenna – i.e., the accelerated charge – moves with a velocity close to the speed of light, c, towards the observer. Then the emission characteristics in the rest system of the antenna are of course unchanged, but the emission in the rest system of the observer has to be transformed according to the rules of Lorentz's transformations. The technically interesting case, which will be considered exclusively in the following, is the case where the velocity $\vec{v}$ is perpendicular to the acceleration $\dot{\vec{v}}$ of the emitting charge (e.g., relativistic charged particles in a transverse magnetic field in a circular particle accelerator): here the relativistic contraction of the emitted radiation is around the direction of maximum emission of the "antenna", while in the case where velocity $\vec{v}$ and acceleration $\dot{\vec{v}}$ are parallel (e.g., in a linear accelerator) the emitted radiation in the forward direction would be zero.

The Lorentz transformation of the nonrelativistic emission [4] leads to a contraction of the emission parallel to the velocity of the charge, which can be approximated for highly relativistic particles of total energy E by

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} \simeq \frac{2e^2\dot{v}^2}{\pi c^3}\,\gamma^6 \cdot \frac{1}{(1+\gamma^2\Theta^2)^3}\left[1 - \frac{4\gamma^2\Theta^2\cos^2\phi}{(1+\gamma^2\Theta^2)^2}\right] \tag{2.4}$$

and to an enhancement by a factor γ^4 with $\gamma = E/mc^2$ of the emitted power P compared to the nonrelativistic case as in eq. (2.2):

$$P = \frac{2}{3}\,\frac{e^2\dot{v}^2}{c^3} \cdot \gamma^4. \tag{2.5}$$

The root mean square opening angle of the sharp forward emission cone is

$$\sqrt{\langle\Theta^2\rangle} = \frac{1}{\gamma}. \tag{2.6}$$

If a particle with charge e moves under the centripetal force of a transverse magnetic field $\vec{B}$ on a curved path with radius ρ the emitted synchrotron radiation cone will sweep like a searchlight over the detector of the observer only for the short time

$$\delta t = \frac{4\rho}{3\,c\,\gamma^3}. \tag{2.7}$$

This represents a single short field pulse, the Fourier components of which extend to frequencies of the order of

$$\omega = \frac{3\pi c\gamma^3}{4\rho}. \tag{2.8}$$

In order to characterize the spectrum of synchrotron radiation the term "critical frequency" ω_c,

$$\omega_\mathrm{c} = \frac{3c\gamma^3}{2\rho} = C_{\omega_\mathrm{c}} \cdot \frac{E^3}{\rho}, \tag{2.9a}$$

with

$$C_{\omega_\mathrm{c}} = 3.37 \cdot 10^{18}\,\frac{\mathrm{m}}{\mathrm{s}\cdot\mathrm{GeV}^3} \tag{2.9b}$$

or "critical photon energy" ε_c,

$$\varepsilon_\mathrm{c} = \hbar\omega_\mathrm{c} = C_{\varepsilon_\mathrm{c}} \cdot \frac{E^3}{\rho}, \tag{2.9c}$$

with

$$C_{\varepsilon_\mathrm{c}} = 2.218 \cdot 10^{-6} \cdot \frac{\mathrm{m}}{\mathrm{GeV}^2} \tag{2.9d}$$

has been introduced.

Synchrotron radiation from a bending magnet can approximately be considered as a wave packet from a short electric field pulse, created by the transverse acceleration at the position of the particle. This is visible to the observer only for a short time intervall δt and – due to its sharp time variation at the position of the observer – propagates towards the observer as a packet of electromagnetic waves with frequencies up to ω. The polarization remains parallel to the acceleration.

This mechanism opens the possibility of producing highly focussed (range of mrad's), intense (Watts), linearly polarized radiation down to wavelengths in the sub-micrometer or even nanometer range. It was first exploited in circular, high energy electron accelerators and storage rings by using the radiation created by highly relativistic intense electron beams in the bending magnets of the machines.

The next step to dedicated synchrotron radiation sources was the development of a special arrangement of short bending magnets of opposite polarity, called wiggler (see figure 1).

Due to the linear arrangement of successive bending magnets in a wiggler an observer would receive photons from many dipoles. The wiggler radiation is therefore an overlap of the radiation fans from all of the individual small bending magnet sources of the wiggler. As long as the bending angle ϕ_ID of each of the N individual bending magnets is large compared to the intrinsic opening angle $1/\gamma$ of the synchrotron radiation the wave packets emitted by an individual electron in the different poles do not interfere. Therefore the radiation received by the observer is just a superposition of the radiation fans of all of the individual small bending magnets and therefore the intensity received is roughly N times that from a bending magnet source. In contrast to the bending magnet source, which uses a magnetic element, which is anyhow necessary in the magnet structure (lattice) of an accelerator, the wiggler was the first magnetic device that was specially constructed to enhance the intensity and had to be inserted additionally into the original magnet lattice of the accelerator. This special type of magnetic device, which is constructed to improve special characteristics of synchrotron radiation, is called "insertion device".

The most important insertion device is the so called "undulator". It is a similar arrangement of successive small bending magnets like the "wiggler" but with the special feature that the bending angle ϕ_ID of the individual magnet pole is reduced to values small compared to the intrinsic opening angle $1/\gamma$ of the synchrotron radiation. Therefore, the electron moves along the undulator always together with the electromagnetic fields created by the same electron in the poles already passed. It is

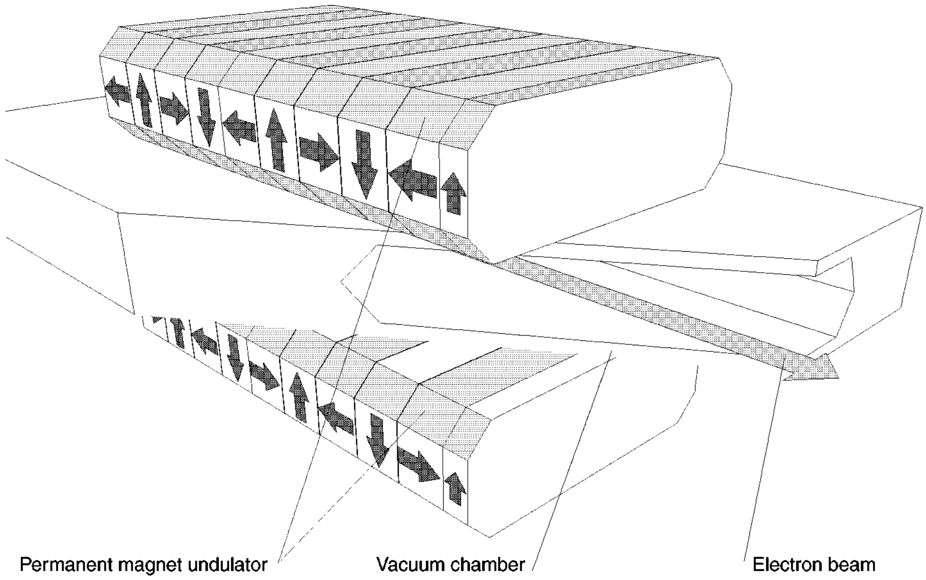

Figure 1. Scheme of the magnet structure of an insertion device, undulator and wiggler, respectively [5]. The electron beam is traveling in a vacuum chamber which is between the magnet structure of the insertion device. The structure can be moved up and down in order to adjust the magnitude of the magnetic field acting on the electron beam in order to vary the emitted X-ray energy.

therefore essential to know whether the electric field $\vec{E}$ created in one distinct pole is in phase with the electric fields created in the poles already passed. Let us assume for a moment that the electron would move along the undulator precisely with the speed of light: then the electric field $\vec{E}$ created in one pole would be precisely compensated by the electric field $-\vec{E}$ created in the following pole of opposite polarity and therefore opposite acceleration and no synchrotron radiation would be produced at all. But in reality the electrons move with a velocity smaller than the speed of light.

This creates a phase shift between the created field and the successive accelerations of the electrons in the successive magnetic poles such that one can always find a wavelength λ in the wave packet of the synchrotron light for which the successive accelerations in the successive poles and therefore the electric fields would be in phase. For these λ's (and their multiple harmonics) the fields created in the different poles of the undulator would constructively interfere, while – depending on the number of poles in the undulator – the effect of other wavelengths would more or less cancel.

3. Single particle emission characteristics

3.1. *Bending magnet radiation*

Bending magnet radiation is characterized by the fact that an observer sees radiation only from one very short section of a particle's trajectory created from one single electric field pulse.

The angular distribution of the synchrotron radiation emitted from a single particle is given by

$$\frac{\mathrm{d}\mathcal{F}_B(\omega)}{\mathrm{d}\Omega} = \frac{3\alpha}{4\pi^2}\,\gamma^2\,\frac{\Delta\omega}{\omega}\,\frac{I}{e}\,\frac{\omega^2}{\omega_\mathrm{c}^2}\big(1+\gamma^2\Theta^2\big)^2\left[K_{2/3}^2(\xi)+\frac{\gamma^2\Theta^2}{1+\gamma^2\Theta^2}\,K_{1/3}^2(\xi)\right], \qquad (3.1)$$

where K is a modified Bessel function of the second type with

$$\xi = \frac{1}{2}\frac{\omega}{\omega_\mathrm{c}}\left[\sqrt{1+\gamma^2\Theta^2}\right]^3,$$

ϕ and Θ are the observation angles in horizontal and vertical directions, respectively, and α is the fine structure constant.

3.2. *Radiation from insertion devices*

Radiation from Insertion Devices (IDs) is characterized by the fact that an observer sees synchrotron radiation from several short bending magnets arranged such that the magnetic field varies sinusoidally

$$B(x) = B_0 \cdot \cos(2\pi x/\lambda_u), \qquad (3.2)$$

where B_0 is the peak magnetic field of one of the short bending magnets and λ_u is the magnetic period. To which extent the radiation emitted from the different poles is able to interfere depends on the ratio of the intrinsic opening angle ($1/\gamma$) to the bending angle (ϕ_ID) of a single pole, and is usually characterized by the deflection parameter K,

$$K = \phi_\mathrm{ID}\cdot\gamma = \frac{eB_0\lambda_u}{2\pi\, mc} = 0.934\cdot\frac{\lambda_u}{\mathrm{cm}}\cdot\frac{B_0}{\mathrm{T}}. \qquad (3.3)$$

If $K \ll 1$ the radiation shows a strong interference effect (undulator) while with $K \gg 1$ the radiation from the different poles overlaps incoherently (wiggler).

In the case of an undulator the wavelength of the fundamental, λ_1, on axis ($\Theta = \phi = 0$) is given by

$$\lambda_1 = \lambda_u\,\frac{(1+\frac{1}{2}K^2)}{2\gamma^2}. \qquad (3.4)$$

The relative bandwidth $\delta\lambda/\lambda$ of the nth harmonic depends on the number of poles N and is given by

$$\frac{\delta\lambda_n}{\lambda_n} \cong \frac{1}{nN}. \qquad (3.5)$$

The on axis peak intensity of the nth harmonic is given by

$$\left.\frac{\mathrm{d}\mathcal{F}_n}{\mathrm{d}\Omega}\right|_0 = \alpha N^2\gamma^2\,\frac{\Delta\omega}{\omega}\,\frac{I}{e}\,F_n(K), \quad n = 2k+1,\ k\in\mathbb{N}, \qquad (3.6a)$$

$$\left.\frac{\mathrm{d}\mathcal{F}_n}{\mathrm{d}\Omega}\right|_0 = 0, \quad n = 2k,\ k\in\mathbb{N}, \qquad (3.6b)$$

with

$$F_n(K) = \frac{K^2 n^2}{(1+\frac{1}{2}K^2)^2}\left\{J_{\frac{n-1}{2}}\left[\frac{nK^2}{4\,(1+\frac{1}{2}K^2)}\right] - J_{\frac{n+1}{2}}\left[\frac{nK^2}{4\,(1+\frac{1}{2}K^2)}\right]\right\}^2. \tag{3.6c}$$

Here the J's are Bessel functions.

The angular distribution of the nth harmonic is approximately given by

$$\sigma_{r'} \cong \sqrt{\frac{\lambda_n}{L}} = \frac{1}{\gamma}\sqrt{\frac{1+\frac{1}{2}K^2}{2Nn}} \tag{3.7}$$

with $L = N\lambda_u$ the length of the undulator.

4. Emission from real accelerator beams

Up to now only the emission from a single particle has been considered. In real accelerator beams, which consist of a large number of individual uncorrelated particles, the emission characteristics are modified because

- not all particles of the beam follow the trajectory exactly (finite emittance),
- not all particles have exactly the same energy (finite energy spread),
- the flow of particles in a beam is not continuous.

4.1. Emittance effects

In a circular accelerator the external magnetic guide field (magnet lattice) determines only one trajectory on which a particle of the nominal energy E can travel stably on a closed path. This trajectory is called the “closed orbit”. But this trajectory is not the only possible stable trajectory: due to the focusing properties of the lattice beam particles can also travel on trajectories slightly deviating from the closed orbit in angle x', y' and position x, y. If one plots all possible stable trajectories by their horizontal and vertical deviation $\Delta x'$, $\Delta y'$ and Δx, Δy from the closed orbit at a given longitudinal position in the accelerator the resulting area in phase space is called the acceptance of the accelerator (figure 2).

If trajectories outside the acceptance are blocked by geometrical obstacles (like the walls of the vacuum chamber) or if the particles travelling on trajectories outside the acceptance are unstable due to nonlinear field components in the magnetic guide fields (e.g., the sextupole components necessary to reduce chromatic errors of the lattice) one talks of geometrical and dynamic acceptance, respectively.

In a similar way a given lattice may provide stable trajectories not only for the “on-energy” particles (having precisely the nominal energy E) but also for particles with energies slightly deviating from E. The ability to accept particles with different energy is called the energy acceptance.

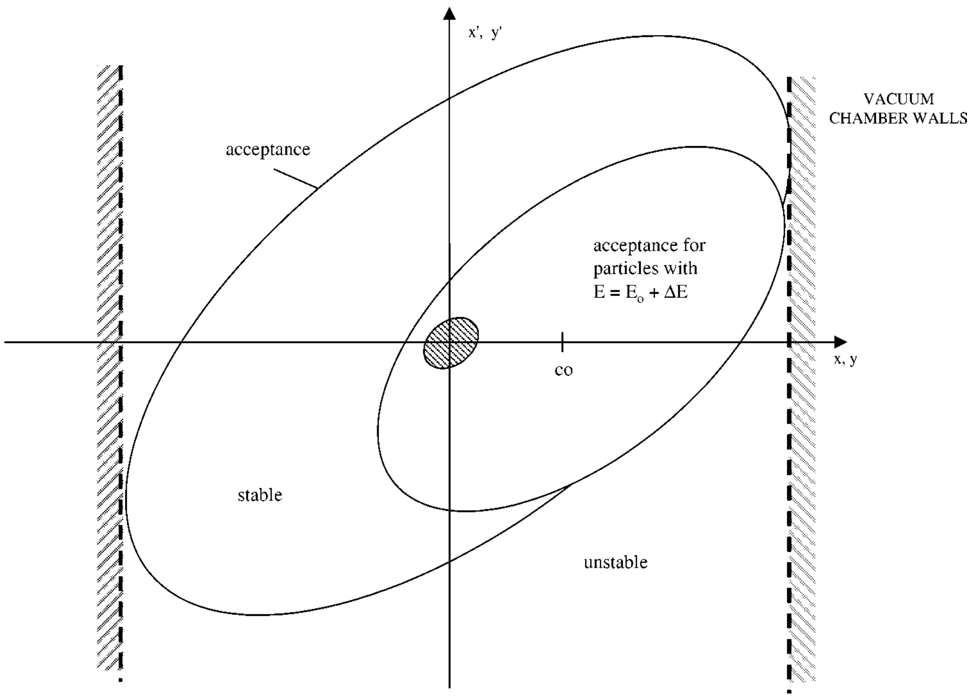

Figure 2. Acceptance and emittance. The acceptance of an accelerator is shown by the big ellipse. The walls of the vacuum chamber or nonlinear field components may reduce this acceptance. The smaller ellipse gives the acceptance for particles on a closed orbit (co) with an energy different from the nominal one. Particles with nominal energy travel in the phase space given by the dashed ellipse (emittance) with the closed orbit located in the origin.

These acceptances define the maximum deviations in position, angle and energy from the on-energy closed orbit for a particle to be stable in the lattice of the accelerator. Real beams in synchrotron light sources normally occupy only a small fraction of the acceptance region. The distribution of particles in phase space and in energy depends on their equilibrium between the excitation of amplitudes around the closed orbit due to the emission of synchrotron light quanta (quantum excitation) and the damping mechanism, and is therefore normally of Gaussian shape.

Each beam particle can be represented by a point in the phase space diagram. The area enclosed by a particle density line at one σ from the peak of the density distribution is called the emittance of the beam and characterizes size and divergence of the circulating particle beam.

In an ideal beam with zero emittance all particles travel on a closed orbit. The cross-section of this beam is pointlike and its divergence is zero. In this ideal case the synchrotron radiation from a given point shows the characteristics of single particle synchrotron light emission. The divergence is governed by eq. (3.7) and the point dimensions are called diffraction-limited (dl).

Point size $\sigma_{\rm dl}$ and minimum divergence $\sigma'_{\rm dl}$ for light of wavelength λ are given by

$$\sigma_{\rm dl} = \frac{\sqrt{L\lambda}}{4\pi}, \tag{4.1a}$$

$$\sigma'_{\rm dl} = \sqrt{\frac{\lambda}{L}} \tag{4.1b}$$

with L the apparent axial extension of the source. This apparent axial extension could be either the undulator length in the case of an undulator source or the particle's path length needed for a deflection of $1/\gamma$ in a bending magnet source. In such a case of a "diffraction limited synchrotron light source" the phase space occupied by the synchrotron light beam (photon emittance) would be

$$\varepsilon_{\rm dl} = \sigma_{\rm dl} \cdot \sigma'_{\rm dl} = \frac{\lambda}{4\pi}. \tag{4.2}$$

The photon emittance determines the minimum spot size that can be reached by the beam line imaging optics. Since in many advanced synchrotron light experiments the ability to reach micrometer or even sub-micrometer image sizes is crucial nearly all modern synchrotron light sources use the flux of photons per photon emittance as their main figure of merit ("brilliance"). Unfortunately, all real beams of particle accelerators have finite emittances. Therefore, the effective photon emittance $\varepsilon_{Tx}, \varepsilon_{Ty}$ is usually larger than given by eq. (4.2). The effective photon emittance can be calculated by folding the single particle photon emittance $\varepsilon_x, \varepsilon_y$ and the electron beam emittance ε_r. Figure 3 shows the folding of the diffraction limited photon emittance with the electron beam phase space in case of the storage ring of the Swiss Light Source (SLS) [6].

The form and orientation of the ideal photon phase space is governed by the photon wavelength. For a given accelerator they can normally not be freely chosen. However, the form and orientation of the electron beam phase space can be matched (or at least optimized) to the emission characteristics of bending magnets, wigglers and undulators by an appropriate choice of the focusing properties of the lattice of the accelerator[1].

4.2. *Effects due to the energy spread in the electron beam*

Due to the stochastic emission of synchrotron light quanta and the subsequent restitution of energy by the accelerating cavities the beam particles show a Gaussian

[1] The dimensions for the electron beam may be calculated in first approximation by the following expressions, with ε_i the emittance and β_i the β-function of the lattice ($i = x, y$): $\sigma_i = \sqrt{\varepsilon_i \cdot \beta_i}$, $\sigma'_i = \sqrt{\varepsilon_i/\beta_i}$, and for the effective X-ray beam size by convoluting these quantities with the diffraction limited values $\sigma_{\rm dl}$, $\sigma'_{\rm dl}$ ($\varepsilon_r \equiv \varepsilon_{\rm dl} = \sigma_{\rm dl} \cdot \sigma'_{\rm dl}$, see eq. (4.2)) of the X-ray beam: $\sigma_{T_i} = \sqrt{\sigma_i^2 + \sigma_{\rm dl}^2}$, $\sigma'_{T_i} = \sqrt{\sigma_i'^2 + \sigma_{\rm dl}'^2}$.

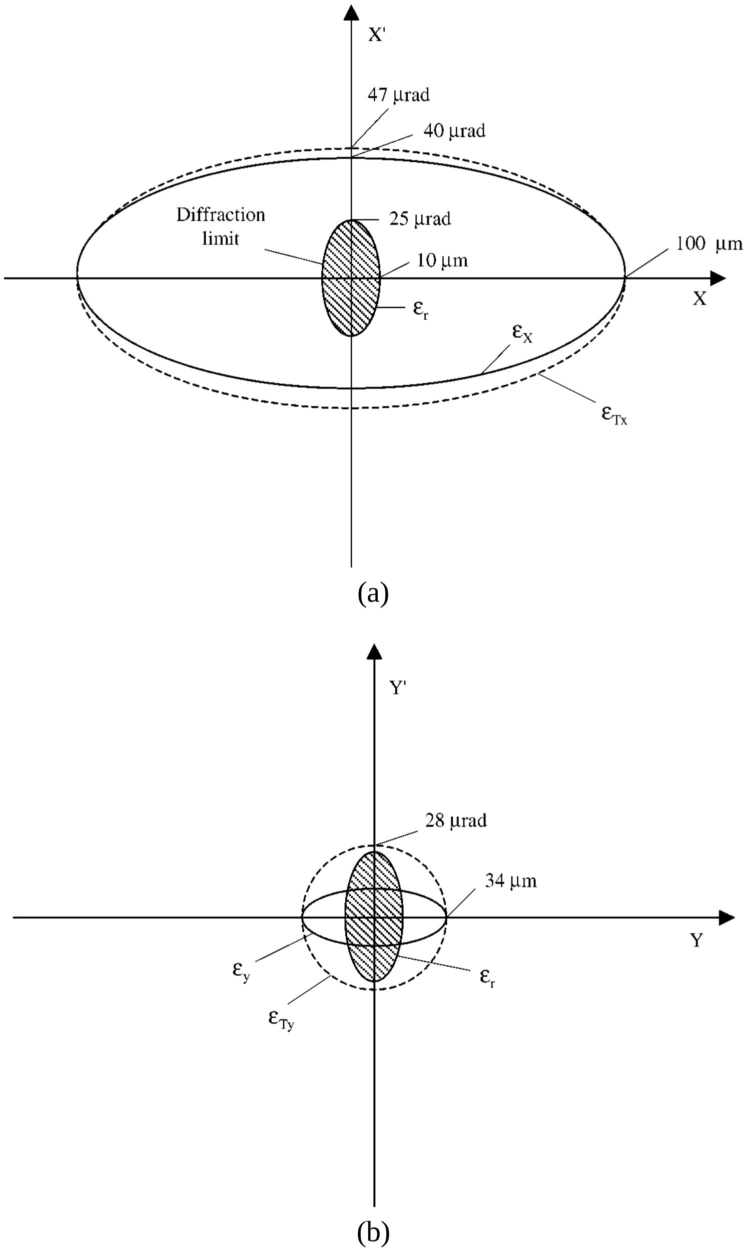

Figure 3. Folding of the diffraction limited photon emittance $\varepsilon_r \equiv \varepsilon_{\mathrm{dl}}$ with the electron beam phase space ε_x (a) and ε_y (b) gives an effective photon emittance ε_{Tx} (a) and ε_{Ty} (b), respectively. As an example values are taken from SLS [6] with $\varepsilon_r = 0.25$ nm, $\varepsilon_x = 4$ nm, $\varepsilon_y = 0.4$ nm, $\varepsilon_{Tx} = 4.7$ nm, $\varepsilon_{Ty} = 1$ nm and undulator length $L = 5$ m, particle energy $E = 2.1$ GeV, photon energy/wavelength $\varepsilon = 400$ eV$/\lambda = 3$ nm and a β-function with $\beta_x = \beta_y = 2.5$ m. The ellipses give the dimensions of the beams in the phase space (spatial (x, y) and angular (x', y'), respectively) in units of one σ [7].

energy distribution around the nominal energy E with a standard deviation of

$$\sigma_\varepsilon = E \cdot \gamma \sqrt{\frac{3.83 \cdot 10^{-13}\,\mathrm{m}}{J_\varepsilon \cdot \rho}} \tag{4.3}$$

with J_ε the damping partition number for energy oscillations, which in usual machines is close to 2.

The energy distribution, σ_ε/E, usually has a relative width of 10^{-3}–10^{-4} if the beam is operated below all longitudinal instability levels as is usual in modern synchrotron light sources. If the beam is operated above a longitudinal instability level the energy distribution will become time dependent and will increase in extreme cases up to the energy acceptance level of the accelerator, which can have a width of a few times 10^{-2}.

The finite energy distribution in the electron beam increases the width of the spectrum of the emitted synchrotron light. Since in bending magnet sources and in wigglers the single particle spectrum is much wider than the energy spread of the emitting electrons, the energy spread of the electrons is of no practical importance. But low-K undulators produce photon spectra with narrow energy widths in the harmonics. The highly desired narrowness of the undulator spectrum can be widened by the unavoidable energy spread of the particle beam or even completely spoiled when the particle beam is operated above the longitudinal instability level. Figure 4

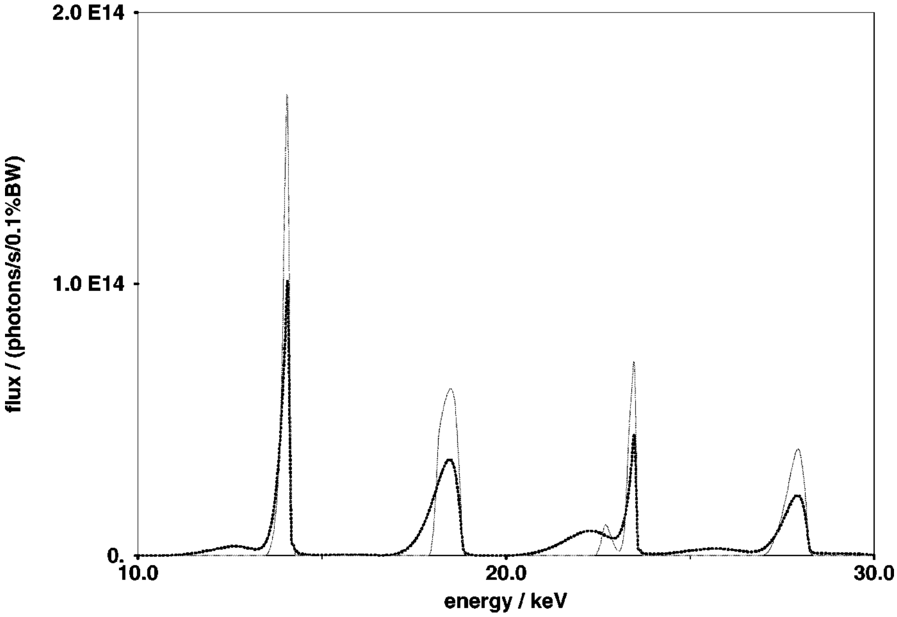

Figure 4. Undulator spectrum with (thick dotted line) and without (thin solid line) finite energy spread in the particle beam. As an example the flux in the central cone of a standard ESRF undulator (1.65 m long) with 32 mm period and 12.5 mm magnet gap at 6 GeV and 100 mA has been calculated. Then the intensity of the third harmonics is peaked at 14.4 keV the ^{57}Fe resonance energy.

shows an undulator spectrum with and without finite energy spread in the particle beam.

4.3. Time structure

Particles circulating in an accelerator lose energy due to the emission of synchrotron radiation and to parasitic losses in the vacuum chamber. In order to keep the particles in the time average on the nominal energy the lost energy is resupplied to the particle by an alternating accelerating electric field. The accelerating field is usually provided by one or more cavities resonating at frequencies in the UHF domain.

The average energy δE lost by an electron traveling one turn around a circular accelerator is given by integrating the radiation power (eq. (2.5)) along the circumference

$$\delta E = C_\gamma \cdot \frac{E^4}{\rho} \tag{4.4a}$$

with

$$C_\gamma = \frac{4\,\pi}{3}\,\frac{r_e}{(mc^2)^3} = 8.86 \cdot 10^{-5}\,\frac{\mathrm{m}}{\mathrm{GeV}^3}, \tag{4.4b}$$

where E is the electron energy, ρ the bending radius of the accelerator and r_e the classical electron radius. In addition, losses in the insertion devices and parasitic losses have to be considered. In order to supply the lost energy by the accelerating cavity the electron must pass the accelerating cavity at the average phase angle ϕ_s, where the voltage U_{ϕ_s} is just sufficient (figure 5):

$$eU_{\phi_s} = \delta E. \tag{4.5}$$

In order to secure the synchronization between the particle traveling around the accelerator on the design orbit and the alternating voltage of the accelerating cavity (which gave the synchrotron its name) the rf frequency f_{rf} of the cavity must be an integral multiple of the fundamental frequency f_0 of the particle:

$$f_{\mathrm{rf}} = m \cdot f_0. \tag{4.6}$$

m is called the harmonic number of the accelerator and characterizes the maximum number of stable phase points on the circumference of the accelerator.

Since the emission of synchrotron light quanta is a stochastic process, the electrons lose not δE but at each turn a slightly different energy with δE as an average. It is one of the essential principles of operation of a synchrotron that particles with small deviations from the nominal energy E are not lost from the synchronization but can stably oscillate around the nominal energy E or the nominal stable phase point ϕ_s (phase stability); this is possible because the orbit length C and therefore also the fundamental frequency depends on the particle energy by

$$\delta C/C = \alpha_{\mathrm{comp}} \cdot \delta E/E. \tag{4.7}$$

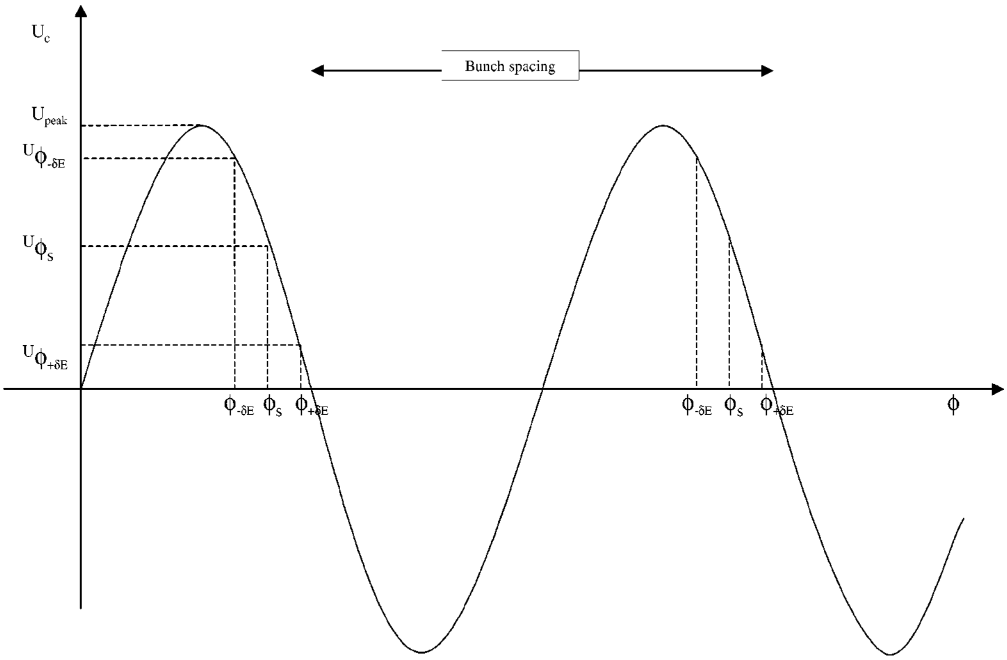

Figure 5. Acceleration voltage U_c and range of synchrotron oscillations between $\phi_{-\delta E}$ and $\phi_{\delta E}$ around the stable phase angle ϕ_s. In order to compensate the losses due to synchrotron radiation emission the electrons have to pass the cavity at an average phase angle ϕ_s giving an accelerating voltage U_{ϕ_s}. The wavelength of U_c is given by the bunch spacing.

α_{comp} is called the momentum compaction factor. A particle with slightly higher than the nominal energy E will usually travel on a slightly longer orbit outside the closed orbit. Consequently, it will arrive at the cavity at a slightly later phase $\phi_{\delta E}$ seeing a slightly smaller accelerating voltage $U_{\phi_{\delta E}}$ than the nominal particle. It will, therefore, gain less energy than the nominal particle, which in turn will move the particle's energy back to the nominal energy E. These particles will therefore oscillate around the stable phase point ϕ_s and around the nominal energy E like in a harmonic potential. The oscillations are called synchrotron oscillations, the potential in which the particles oscillate is called a bucket.

From figure 5, it is evident that the maximum possible stable phase amplitude of the synchrotron oscillation depends on the ratio of the peak cavity voltage U_{peak} and the voltage at nominal phase U_{ϕ_s}. This ratio is called the overvoltage factor q. An energy oscillation is associated with the phase oscillation. The corresponding maximum possible energy deviation δE_{max} is given by

$$\delta E_{\text{max}} = E \sqrt{\frac{e\,U_{\phi_s} \cdot F(q)}{\pi \cdot m \cdot \alpha_{\text{comp}} \cdot E}} \tag{4.8}$$

with

$$F(q) = 2 \cdot \left\{ \sqrt{q^2 - 1} - \arccos \frac{1}{q} \right\}. \tag{4.9}$$

Since the particles change their energies stochastically due to the emission of synchrotron light quanta synchrotron oscillations are permanently excited. On the other hand, there exists a damping mechanism for transverse and longitudinal particle oscillations in all circular accelerators. The equilibrium between excitation and damping of synchrotron oscillations leads to a distribution of particles around the stable phase point, which is normally Gaussian if the beam is operated below any instability level and if the accelerating voltage is large compared to the average energy loss. The natural width of this distribution is given by

$$\sigma_l = \frac{\alpha_{\text{comp}} \cdot c}{2\pi f_0} \cdot \frac{\sigma_\varepsilon}{E}. \tag{4.10}$$

Despite the fact that in synchrotron light sources the natural bunch length is small compared to the bucket length (or, in other words, the natural distribution of energies in a bunch is small compared to the maximum acceptable energy deviation) it should be noted that the Gaussian tails fill the available bucket. Outside the buckets only unstable particles can exist for short times.

If the frequency f_0 of the accelerating cavities is chosen such that one period T_0 of the rf field just equals the time needed by one particle to travel once around the circumference C of the accelerator,

$$1/f_0 = T_0 = C/c, \tag{4.11}$$

then all particles will be "bunched" around the phase point where the energy gain from the accelerating field just equals the average energy loss due to synchrotron radiation. Therefore there will be just one "bunch" of particles traveling round the circumference of the accelerator in phase with the accelerating field in the cavities. Most accelerators, however, use frequencies f_m,

$$f_m = m \cdot f_0, \tag{4.12}$$

which are harmonics of f_0. The corresponding accelerator has, therefore, m buckets available on the circumference of the accelerator which can be filled or not with electrons by the injector. In order to fill only one of the available buckets, the injector has to deliver electrons only during a length of time which is shorter than or equal to the distance between two adjacent buckets. With frequencies of the accelerating cavities in modern synchrotron light sources of a few hundred Megahertz this means pulse lengths in the order of nanoseconds for the high voltage electron guns of the injector. The gun electronics (or the lasers in case of a photo-electron gun) can usually deliver such an opening pulse, but injector linacs tend to create, during the linac rf power pulse (i.e., during a few microseconds), dark current electrons with intensities of 10^{-4}–10^{-6} with respect to the short main pulse. These electrons could be captured in buckets which are intended to be left completely empty. For special experiments, necessitating cleaner single or few bunch operation, special "cleaning procedures" are employed after injection. These procedures allow one to reach intensity ratios between the intentionally populated bunch and the buckets considered to be empty in the 10^{-7}–10^{-8} range. Figure 6 shows a "single

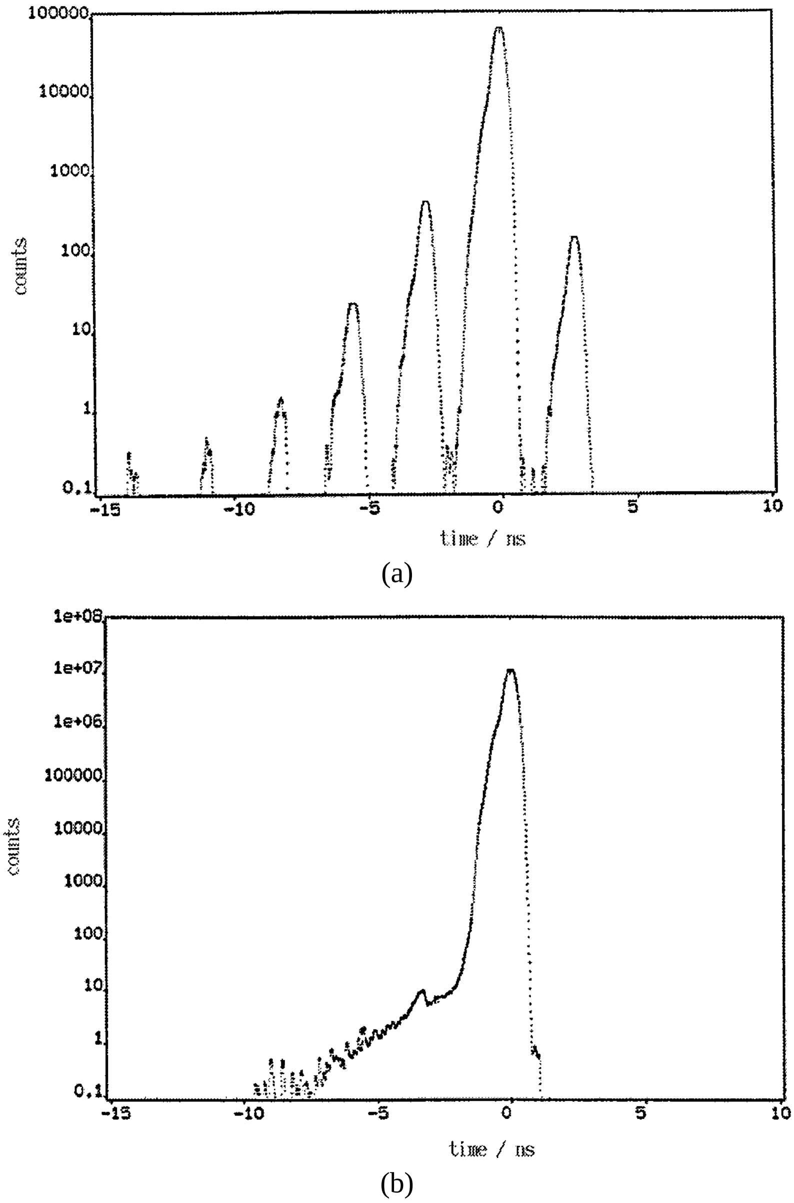

Figure 6. Intensity distribution of a single bunch before (a) and after (b) the cleaning procedure at ESRF [10]. Spurious populated buckets are cleaned better than 10^{-7} with respect to the main bunch. The residual intensity in (b) is due to detector noise.

bunch" with its neighbouring buckets before and after application of cleaning procedures.

5. Collective radiation

In the case that there would be only one single particle in the beam the intensity and the spectrum of the emitted synchrotron radiation would be as described under single particle emission (eqs. (3.1), (3.6a)). If there would be a continuous stream of *n* particles in the beam the emitted spectrum remains the unchanged single particle emission spectrum but the intensity would multiply by n. However, both spectrum and intensity change as soon as the beam gets an ac-component, i.e., as soon as the beam gets bunched or otherwise modulated.

This can be made plausible by a "gedankenexperiment": let us assume that we would calculate the synchrotron emission from an accelerator where we would put all n electrons of the beam into one stiff pointlike cluster of electrons and let this pointlike cluster of n electrons travel along the accelerator. Because this cluster is stiff and pointlike it would behave as a single particle of charge $n \cdot e$. Since the charge appears squared in the intensity formulas the n particles of the cluster would emit an intensity n^2 times the intensity of the same number of particles in a cw continuous beam. Such an emission, where all charges move fully coherent and therefore create precisely the same electric fields, is called coherent emission. Since the number n of particles in a cluster can easily reach 10^9–10^{13}, it is clear that the emitted synchrotron light power would be dramatically increased.

In usual synchrotron light sources the electrons are not concentrated into a "Dirac function" but they are naturally bunched with bunch lengths around centimeters to fractions of millimeters. For those Fourier components of the created electric fields which have wavelengths comparable to or longer than the bunch length the electrons move practically coherently. For these synchrotron radiation wavelengths the emitted power would indeed go with the number N_b of particles in a given bunch squared.

Unfortunately, these very long wavelength components of the emitted synchrotron radiation cannot propagate in the conducting vacuum chamber of the accelerator and therefore these long wavelength enhancements of the spectrum are difficult to detect.

On the other hand, it is technically possible to create a density modulation at optical or even X-ray wavelengths by letting the bunch interact with a very strong optical or X-ray wavefield. It is beyond the scope of this introduction to describe the function of the various mechanisms (optical klystrons (OK), free electron lasers (FEL) or the self amplified stimulated emission (SASE)) which have been invented in order to exploit the fantastically increased spectral power connected with the coherent movement of electrons. But in order to give a flavor of the possibilities of such devices for coherent synchrotron radiation the parameters of a SASE-FEL project at DESY are given below (see [9]).

Table 1

Some relevant parameters of synchrotron radiation sources where nuclear resonance scattering experiments may routinely be carried out.

Parameter	ESRF				APS		SPring-8		DORIS III			PETRA II	
energy [GeV]	6.04				7		8		4.45			12	
particles	electron				electron		electron		positron			positron	
current [mA]	200				100		100		150			50	
circumference [m]	844				1104		1436		289.2			2304	
frequency [MHz]	352.2				351.927		505.58		500			500	
period [ns]	2800				3683		4790		964			7680	
number of buckets	992				1296		2436		482			3840	
bunch length [ps]	50–120				65		100						
modes of operation	$2 \cdot \frac{1}{3}$	16-bunch	single	hybrid	singlet[1]	triplet[2]	$2/3$	21-bunch[3]	1-bunch	2-bunch	5-bunch	14-bunch	42-bunch
spacing [ns]	2.84	176	2800	900	148	102	1.98	228	964	480	192	480	96
lifetime [h]	50	10	5	35	20	40	65	7-12	15	20	25		
max. current [mA]	200	90	15	200	100	100	70	20-60	50	80	150		50
availability [%]	70	25	5	On request	15	85	50	50	On request	25	75	On request	On request
emmitance hor. [nm · rad]	3.9				6.5		6.8		404			25	
emmitance ver. [nm · rad]	0.039				0.06		0.0068		6			0.75	
energy spread [%]	0.1				0.1		0.1		0.11			0.075	
Source properties	ID 18		ID 22N		3 ID		BL09XU[4]		F4	BW4			
size hor. (rms) [μm]	395				300		410		1200	1200		1170	
size ver. (rms) [μm]	9.9				45		35		360	240		134	
div. hor. (rms) [μrad]	10.5				24		17		400	593		21	
div. ver. (rms) [μrad]	3.9				3		3.9		17	25		5.6	
X-ray properties[5]													
source type[6]	U (1.6 m)		U (1.6 m)		U (2.4 m)		U (4.5 m)		BM	W (2.7 m)		U (4 m)	
size hor. (fwhm) [mm]	1.5				2.4		1.9		20	6.6		3.4	
size ver. (fwhm) [mm]	0.8				0.64		0.6		2	0.8		0.8	
div. hor. (fwhm) [μrad]	30.2				58		41		–	4 200		50.5	
div. ver. (fwhm) [μrad]	19.7				15.8		13.8		–	200		17.0	
purity	10^{-9}				10^{-8}		10^{-8}						

[1] 6 adjacent buckets (15% of beam), a space of 156 ns, and a single bucket repeated 22 times with 148 ns between singlets.
[2] 6 adjacent buckets (10% of beam), a space of 190 ns, and 3 adjacent buckets repeated 25 times with 102 ns between triplets.
[3] Under development: either real 21-bunch mode or 21 times 3 adjacent buckets. [4] Other beamlines plan also to do NRS experiments.
[5] At 14.4 keV and at 40 m downstream of source. [6] U – undulator, W – wiggler, BM – bending magnet and length.

TESLA-FEL (at DESY)

Spectral range of coherent radiation	1.5–0.1 nm.
Peak spectral brilliance	10^{33}–10^{34} $\frac{\text{photons}}{\text{s}\cdot\text{mm}^2\cdot\text{mrad}^2\cdot 0.001\,\text{BW}}$.
Pulse length (rms)	80 fs.
Pulse separation	tunable from a few nsec to 1 msec.

6. Parameters of synchrotron light sources presently used for hyperfine interactions

Table 1 gives examples for the parameters of synchrotron light sources presently used for nuclear resonance scattering. The parameters listed are those of 1998. Some of the parameters like “vertical emittance”, vertical source size and X-ray properties depend on different methods of measurement and should therefore be compared with caution.

The indicated modes of operation show that the maximum total beam current depends on the filling pattern and the bunch length. This is due to beam instabilities at higher currents or to the excessive mirror currents in the walls of the vacuum chambers (leading to local heating and successive vacuum deterioration). Especially the single or few bunch modes show – even at the indicated quite moderate total beam currents up to 150 mA – electron beam peak currents within one bunch of a few hundred amperes. This leads to rf wall currents which heat up especially the rf contacts of the rf liners of the vacuum system. In addition, the increase of bunch current is usually accompanied by an increase in bunch length and energy spread. The increased energy spread leads to an increase of the line width of the undulator spectrum with a corresponding decrease in spectral brilliance. An introduction to the wide field of beam instabilities can be found in the book of Winick [7].

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