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Origin of Anomalous Surface Reflection of X Rays*

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The anomalous surface reflection of x rays from "clean" mirrors, reported by Yoneda, is shown to be primarily due to the horizontal angular divergence of the incident beam itself.

The specular reflection of x rays, and diffraction effects, from "smooth" surfaces have been more or less carefully studied but no apparently nonspecular reflection was observed until Yoneda[1] reported the presence of what he called "anomalous surface reflection" (ASR). For a glancing angle of incidence greater than the critical angle, he observed not only the equiangular reflection but also another reflection maximum at an angle smaller than the angle of incidence.

An interpretation of this anomalous reflection has been recently suggested by Warren and Clarke.[2] This is based essentially on the small-angle scattering from "dirt" on the surface of the mirror. However, it appears doubtful that, with an ostensibly clean smooth surface the intensity of small-angle scattered x rays from very small traces of "dirt" or other roughness on the mirror would be sufficiently great to give rise to the observed ASR intensities.[3]

In work here reported the origin of ASR from "clean" mirrors is traced primarily to the horizontal[4] angular divergence of the incident beam itself. For recording the ASR the angle of incidence of the central ray is set to be greater than, but not far greater than, the critical angle. The horizontal divergence is such that the low-angle part of the incident beam strikes the mirror at angles less than the critical angle for which the coefficient of reflection is rather suddenly large. The observed variation of reflected intensity with angle is in the product of the incident-beam intensity and the reflection curve. The graph of the reflected intensity shows the ASR at (or just below) the critical angle where the sudden rise of reflectance overcompensates the lowering of intensity in the wings of the incident beam. ASR indeed shows a sharper cutoff on the high-angle side than on the low-angle side.

In the present experiments the x-ray source was a line-focus copper target tube (Machlett A2L). A narrow beam of monochromatic x rays (CuKα_1; $\lambda=1.537$ Å) was obtained by Bragg reflection from a calcite crystal in conjunction with two slits 0.05 and 0.025 mm wide, respectively. The system gave an incident beam of nominal geometrical divergence of about 0.25 mrad but in fact contained more extensive wings as discussed below. The first mirrors used were polished soft glass; later mirrors used were of a copper film (1000 Å thick) vacuum-deposited on the glass surface. The angular distributions of the incident and reflected beams were recorded with both a proportional-counter assembly and photographically. The photographic film was placed at a distance of 900 mm from the mirror axis and the exposure times for recording the ASR varied from 1–62 h with the x-ray tube operating at 22 kV and 16 mA. A typical photograph is reproduced in Fig. 1, the mirror in this case being copper-on-glass and the exposure 62 h. The angle of incidence for this photograph was 7.5 mrad. (The "additional maxima of intensity" near the specularly reflected beam are discussed later.)

A long-exposure photograph of the direct beam shows that measurable intensity does exist in the wings, as much as 4 mrad on either side of the peak. The approximate shape of the incident beam, as actually measured, is shown in Fig. 2. Of course, the major part of the direct beam is enclosed in a narrow width as dictated by the nominal geometrical divergence. As a further check on the divergence, the mirror was arranged so that the shadow of its edge nearer the film was seen in the ASR photograph. Then, for successive photographs the

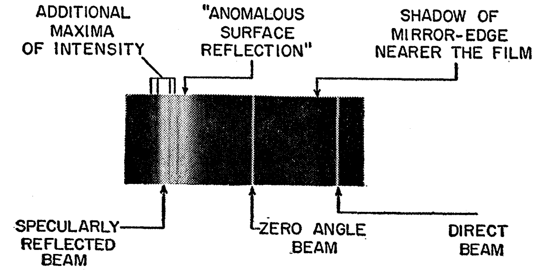

Fig. 1. Photograph with a mirror consisting of a 1000-Å copper film on glass. The glancing angle of incidence = 7.5 mrad (this is the angular distance between the "zero-angle beam" and the "direct beam") and the exposure time = 62 h with the x-ray tube operating at 22 kV and 16 mA.

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[1] Y. Yoneda, Phys. Rev. **131**, 2010 (1963).

[2] B. E. Warren and J. S. Clarke, J. Appl. Phys. **36**, 324 (1965).

[3] M. Renninger [Z. Physik. **100**, 326, (1936)] has observed a type of interference system dependent upon the "dirt" on the surface of the mirror. The reflection maxima of Renninger's system when different than those of the Kiessig type [Ann. Physik **10**, 769 (1931)] were neither observed nor studied in the present work.

[4] Horizontal divergence is measured in the plane perpendicular to the rotation axis of the mirror.

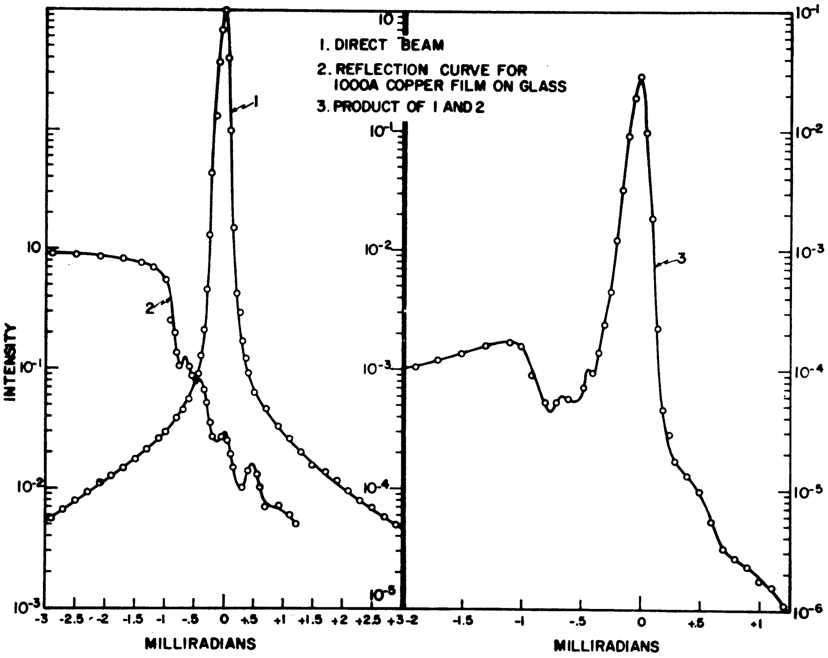

FIG. 2. Semilog graph of (1) direct beam, (2) reflection curve for 1000-Å copper film on glass and (3) the product of (1) and (2) when the angle of incidence is 7.25 mrad.

mirror was slightly shifted parallel to its surface. The observed position of the shadow corresponded each time to the distance of the mirror edge from the central ray and the angle of incidence in just the way the geometrical considerations predict. The widely divergent wings are evidently produced by the scattering of the beam by the closely spaced polished-steel $\frac{1}{16}$-in. cylindrical rods forming the exit slit; the shadow was too sharp to support the contention that the divergence arises from air scattering before incidence on the mirror.

The observed reflected intensity was checked by point-to-point multiplication of the divergent direct beam and the specular-reflection curve for the clean soft glass-mirror. The curve shape was reproduced within the accuracy of measurement. Then to make this check more stringent, several photographs were taken with the mirror consisting of the 1000-Å copper film deposited on the glass. Some maxima and minima of intensity were observed on either side of the normal reflection peak *in addition* to the ASR (as seen in Fig. 1). These correspond very well in position to the maxima and minima in the curve obtained by multiplying the direct beam with the reflection curve for copper film on glass (Fig. 2).

Next, the glass mirror was adjusted to such a position that those portions of the incident beam corresponding to angles of incidence equal to and less than the critical angle would not strike the mirror.[5] A 30-h exposure gave the normal reflection peak as expected but the ASR was just barely visible. The mirror was then shifted parallel to its surface towards the film so as now to include angles of incidence smaller than the critical angle. An equal exposure gave a very much more intense ASR peak.

Although the small-angle-scattering interpretation of Warren and Clarke is a mechanism for producing ASR in the presence of "dirt" on the mirror, a large role is played by the low-intensity wings associated with the incident beam, at least in the present experiments. The "just barely visible" ASR in the previous paragraph is probably due to a trace of "dirt" on the mirror.

It is suggested that the very large intensity of ASR in the case of "clean" glass observed by Warren and Clarke is probably due to a large geometrical divergence of their incident beam, i.e., the nominal divergence, not necessarily the wings. Also it may be noted that in the case of a grossly rough or dirty surface it has been observed[6] that the reflected beam is broader than the one from a smooth surface. The normal peak intensity is reduced at the cost of increased breadth. This contributes to the observed increase in the ratio of ASR to the normal reflection for a rough surface.

In conclusion, all the results of Yoneda,[1] of Warren and Clarke,[2] and of the present measurements, regarding the position and intensity of ASR can be easily explained by a combination of low-intensity wings in the incident beam and by the presence of scattering

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[5] This would be achieved when $\theta > \theta_c(1+x/L)$, where θ is the glancing angle of incidence of the central ray, θ_c the critical angle, L the distance from the mirror axis to an appropriate position between the two slits (very close to the exit slit in the present geometry), and x the distance from the mirror axis to the further edge of the mirror.

[6] N. Wainfan and L. G. Parratt, J. Appl. Phys. **31**, 1331 (1960).

centers on the mirror. The former interpretation no doubt dominates for a clean mirror; the latter may be the more important in the case of a "dirty" mirror.

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Line Shape and Amplitude of Giant Quantum Oscillations in Ultrasonic Absorption

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The line shape of the giant quantum oscillations and the dependence of the amplitude of these oscillations on ultrasonic frequency, magnetic field, and temperature are calculated on the basis of the theory of Gurevich *et al.* Expressions for determining cross sections of the Fermi surface, effective masses, g factors, matrix elements of the electron-phonon interaction, and electron relaxation times, by the use of giant quantum oscillations, are given. Measurements of the line shape and amplitude of the giant quantum oscillations in gallium are presented and compared with theory.

I. INTRODUCTION

WHEN ultrasonic waves are propagated in a pure metal at very low temperatures, the attenuation coefficient may exhibit quantum oscillations as a function of the magnetic-field intensity. Under certain conditions these "oscillations" become a series of sharp absorption peaks separated by wide absorption minima. This series of absorption peaks, which has been termed "giant quantum oscillations" (GQO), was first predicted by Gurevich, Skobov, and Firsov (GSF).[1] An alternative theoretical treatment of this phenomenon has been given by Quinn and Rodriguez[2] and the theory was further extended by several workers.[3–10] The spike-like GQO were first observed in bismuth by Korolyuk.[11] Subsequently the present authors have carried out a quantitative investigation of the line shape of the GQO in gallium for the purpose of determining the effective mass and the g factor of the carriers which give rise to the "oscillations."[9,10] In the present paper the theory of GSF[1] and Gantsevich and Gurevich[3] (GG) is applied to calculate the line shape of the giant absorption peaks for the case of an arbitrary Fermi surface and arbitrary angle, but one which is not too close to 90°, between the direction of the magnetic field and the direction of sound propagation. The dependence of the amplitude of the absorption peaks on ultrasonic frequency, magnetic field intensity and temperature is also considered. Expressions for determining cross sections of the Fermi surface, effective masses, g factors, effective deformation potentials, and electron collision times by use of the GQO are given. Detailed experimental results in gallium are presented and compared with theory. The present discussion is concerned only with longitudinal sound waves. Furthermore, we restrict ourselves to the case of a sufficiently high magnetic field in which the energy separation between Landau levels is large enough to prevent the occurrence of transitions between different Landau levels due to the absorption of a phonon from the sound wave.

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* Supported by the U. S. Air Force Office of Scientific Research.

† Operated with support from the U. S. Air Force.

[1] V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksperim. i Teor. Fiz. **40**, 786 (1961) [English transl.: Soviet Phys.—JETP **13**, 552 (1961)].

[2] J. J. Quinn and S. Rodriguez, Phys. Rev. **128**, 2487 (1962).

[3] S. V. Gantsevich and V. L. Gurevich, Zh. Eksperim. i Teor. Fiz. **45**, 587 (1963) [English transl.: Soviet Phys.—JETP **18**, 403 (1964)].

[4] D. N. Langenberg, J. J. Quinn, and S. Rodriguez, Phys. Rev. Letters **12**, 104 (1964).

[5] M. S. Svirskii, Zh. Eksperim. i Teor. Fiz. **44**, 628 (1963) [English transl.: Soviet Phys.—JETP **17**, 426 (1963)].

[6] S. Rodriguez, Phys. Rev. **130**, 929 (1963).

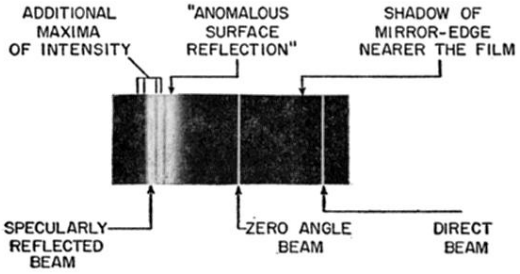
[7] R. F. Kazarinov and V. G. Skobov, Zh. Eksperim. i Teor. Fiz. 43, 1496 (1962) [English transl.: Soviet Phys.—JETP **16**, 1057 (1963)].

[8] V. G. Skobov, Zh. Eksperim. i Teor. Fiz. **40**, 1446 (1961) [English transl.: Soviet Phys.—JETP **13**, 1014 (1961)].

[9] Y. Shapira and B. Lax, Phys. Rev. Letters **12**, 166 (1964).

[10] Y. Shapira, Phys. Rev. Letters **13**, 162 (1964).

[11] A. P. Korolyuk, Fiz. Tverd. Tela **5**, 3323 (1963) [English transl.: Soviet Phys.—Solid State **5**, 2433 (1964)].



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