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# Effect of roughness on the magnetic structure of ferro/ antiferromagnetic interface

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## Abstract

Spin structures at the ferro/antiferromagnetic interfaces perturbed by defects such as atomic high steps are analytically investigated. A two-dimensional model is proposed to describe the spin distribution formed on the interfacial step at the domain wall. A criterion of the domain wall configuration relative to the interface is found, defined by the magnetic and geometrical characteristics of the interface and the magnet. © 2001 Published by Elsevier Science B.V.

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Layered magnetic structures and interfaces between different magnetically ordered media have aroused considerable interest in recent years due to their wide variety of surprising features and a multiplicity of technological uses. The roughness of atomic high steps necessarily abundant on the interface involves severe consequences for the magnetic order of the layered systems. The intention of the present paper is to describe spin structures at ferro/antiferromagnetic (FM/AFM) interfaces perturbed by defects such as steps. We develop a model that allows one to obtain analytical expressions for the magnetic ordering throughout the volume of the system and for the energy of the domain walls (DWs) of various configurations. Information on the real spin

distribution at the perturbed interface expressed in terms of the material parameters of the magnet can be used as a basis for analysis of the observable physical effects, the formation of DWs may lead to, such as exchange bias and other related phenomena.

Consider classical Heisenberg FM/AFM system with atomic high step on the interface, taking into account a weak easy-axis anisotropy  $\gamma$  along the  $x$  direction in the easy  $xz$ -plane (Fig. 1). As it will be seen from below, qualitative analysis of the magnetic structure, we are interested in, is allowed under the assumption of equal anisotropy for FM and AFM, however, the quantitative analysis would require one to differ anisotropy for the two layers. At the exchange interaction through the interface  $J_S$  under a critical value  $J^*$  spin ordering in FM and AFM is ideal, and collinear DW forms along one of the  $x$  half-axes. At  $J^* < J_S < J^{**}$  the DW takes noncollinear form. As  $J_S$  reaches the critical value  $J^{**}$ , the DW is

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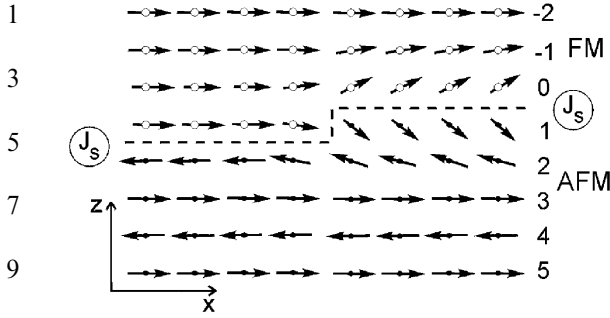


Fig. 1. DW caused by a step on FM/AFM interface given single-ion anisotropy in the easy plane ( $xz$ ).

repelled from the interface since the energy of the DW in the layer is less than that at the interface. To find a criterion of the DW orientation and to determine the values  $J^*$  and  $J^{**}$ , calculate the energy of the noncollinear DW at the step along the interface.

From the energy of the magnetic interaction in the spin chain along the  $z$ -axis at fixed  $x$  static equations for spin deviations  $\varphi$  in the chain can be derived. After variables substitution taking into account “layered” ordering in AFM, linearized equations take the form:

$$J_A b^2 \frac{\partial^2 \varphi}{\partial z^2} + \frac{\gamma}{2} \sin(2\varphi) = 0,$$

$$J_F b^2 \frac{\partial^2 \varphi}{\partial z^2} + \frac{\gamma}{2} \sin(2\varphi) = 0, \quad (1)$$

where  $b$  is lattice parameter along  $z$  direction,  $J_A$  and  $J_F$  are the exchange constants in the  $z$ -directions in AFM and FM. Eqs. (1) is complemented by the boundary conditions

$$bJ_A \left. \frac{\partial \varphi}{\partial z} \right|_{z=-b/2} = J_S \sin(\varphi_0 - \varphi_1),$$

$$bJ_F \left. \frac{\partial \varphi}{\partial z} \right|_{z=+b/2} = J_S \sin(\varphi_0 - \varphi_1). \quad (2)$$

The solutions of Eqs. (1) describe the rotation of spins in a chain along  $z$  at fixed  $x$ :

$$\varphi = 2 \arctan \exp((z_A - z)/l_A), (z < 0),$$

$$\varphi = 2 \arctan \exp((z_F - z)/l_F), (z > 0), \quad (3)$$

where  $l_A = b\sqrt{J_A/\gamma}$  and  $l_F = b\sqrt{J_F/\gamma}$  are the “magnetic lengths” in the half spaces. The values  $z_A$  and  $z_F$  can be defined from the boundary conditions (2) and are the functions of the parameters  $J_A$ ,  $J_F$ ,  $J_S$ ,  $\gamma$ . Using Eqs. (3) we obtain the energy of the unit length of the DW along the interface:

$$E_{\parallel} \approx \sqrt{J_F \gamma} + \sqrt{J_A \gamma} - \frac{\gamma}{2} + \frac{\gamma}{b}(z_F - z_A) + 2J_S \left( \frac{b/2 - z_F}{2l_F} + \frac{b/2 + z_A}{2l_A} \right)^2. \quad (4)$$

Energy (4) appears to be the function of the exchange integrals of FM, AFM and through the interface, depending also on the easy-axis anisotropy parameter. Common expression for the DW energy immediately follows from Eqs. (4) in the case  $J_A = J_F = J_S$ :  $E_0 = 2\sqrt{J\gamma}$ , which agrees with that obtained by the direct calculation of the DW energy in the homogeneous magnet. To compare the energies of variously configured DW, consider the case of equal values of the exchange parameters in FM and AFM:  $J \equiv J_A = J_F \neq J_S$  (we use the assumption of equal exchange constants in FM and AFM to obtain some qualitative results. Note, that, for Fe/Cr, as an example,  $J_{Fe}/J_{Cr} \approx 2$  while  $J_{Fe}/J_{Fe-Cr} \approx 10$  and thus the assumption is valid to be a good approximation). Then, the energy of the unit length of the DW along the interface is

$$E_{\parallel}^0(J, J_S, \gamma) = 2\sqrt{J\gamma} + \frac{\gamma}{2} \left( 1 - \frac{J}{J_S} \right). \quad (5)$$

Comparing this expression with the energy of the collinear DW in the plane of the interface  $E_{col} = 2J_S$ , a critical value of the exchange interaction through the interface  $J_S^*$  can be found, at which the transformation of the collinear DW into the noncollinear DW occurs:  $J_S^* = \frac{1}{2}\sqrt{J\gamma}$ . At  $J_S > J^{**} = J$  the energy of the DW along the interface exceeds the energy of the DW within the thickness of the magnet, the DW is repelled from the interface and is oriented perpendicular to the interface. It is easy to obtain the value of the  $J^{**}$  for  $J_F \neq J_A$ :  $J^{**} = \sqrt{J_F J_A}$ . If the exchange parameters in FM and AFM differ, the DW at

1  $J_S > J^{**}$  forms, obviously, in the magnet with the  
 2 smallest value of the exchange interaction. These  
 3 conclusions as to DW orientation are in agreement  
 4 with the results of numerical calculations for Fe/  
 5 Cr multilayers presented in Ref. [1]. If the finite  
 6 thickness of the FM and AFM layers and the finite  
 7 distance  $L$  between the steps on the interface are  
 8 take into account, a prerequisite to the formation  
 9 of the DW along the interface is  $E_{\parallel}L < E_{\perp}h$ , where  
 10  $E_{\perp} = 2\sqrt{\tilde{J}\gamma}$  ( $\tilde{J}$  is the exchange integral along the  
 11  $x$ -direction). The opposite inequality is the condi-  
 12 tion of the DW formation perpendicular to the  
 13 interface.

14 Analytical description of the nonuniform mag-  
 15 netization distribution caused by a monatomic  
 16 step at the FM/AFM interface can be provided in  
 17 the framework of a simple 2D model proposed in  
 18 Ref. [2] for a system of AFM with the lattice  
 19 dislocation. Consider  $J_S$  value on the interval  $J^*$   
 20  $< J_S < J^{**}$  which corresponds to noncollinear DW  
 21 formation along the interface. For an equivalent  
 22 system of two FM half spaces in contact after  
 23 corresponding variables change, long-wave equa-  
 24 tions for the magnetization distribution take the  
 25 form

$$26 \tilde{J}_A a^2 \frac{\partial^2 \varphi}{\partial x^2} + J_A b^2 \frac{\partial^2 \varphi}{\partial z^2} - \frac{\gamma}{2} \sin(2\varphi) = 0,$$

$$27 \tilde{J}_F a^2 \frac{\partial^2 \varphi}{\partial x^2} + J_F b^2 \frac{\partial^2 \varphi}{\partial z^2} - \frac{\gamma}{2} \sin(2\varphi) = 0, \quad (6)$$

28 where  $a$  is lattice parameter along the  $x$  direction,  
 29  $\tilde{J}_F$  and  $\tilde{J}_A$  are, respectively, the exchange integrals  
 30 in FM and AFM in  $x$ -direction. Nonlinear Eqs. (6)  
 31 can be linearized by replacing single-ion aniso-  
 32 tropy  $E_{\text{an}} = \gamma(1 - \cos^2 \varphi)/2$  with the piecewise  
 33 parabolic function, which is possible when the  
 34 exchange interaction in FM and AFM are of  
 35 the same order of value. Since we are interested in  
 36 the magnetization distribution over distances  
 37 larger than atomic dimensions, replace an interface  
 38 with a step by the ideal boundary, having reversed  
 39 the sign of the exchange interaction through it on  
 40 one side of the step. Complementing the boundary  
 41 condition presenting the density of the effective  
 42 forces acting at the interface

$$43 f_{\pm}(x) = \pm \text{sgn}(x) J_S \sin(\varphi|_{z=+b/2} - \varphi|_{z=-b/2}), \quad (7)$$

44 leads us to the following solution of the volume  
 45 problem (6):

$$46 \varphi(x, z > 0) = - \frac{J_S}{\pi a \sqrt{\tilde{J}_F J_F}} \quad 51$$

$$47 \times \int_{-\infty}^{+\infty} dx' K_0 \left( \sqrt{\frac{(x-x')^2}{\sigma_x^2} + \frac{z^2}{\sigma_z^2}} \right) \quad 53$$

$$48 \sin(\chi(x')) \text{sgn}(x') \quad 55$$

$$49 \quad (8) \quad 57$$

50 (and the analogous expression for AFM half  
 51 space), where Macdonald's function  $K_0(k)$  is the  
 52 Green's function of the Klein-Gordon equation;  
 53  $\chi = \varphi|_{z=+b/2} - \varphi|_{z=-b/2}$  the function of relative spin  
 54 deviation at the interface;  $\sigma_x = a\sqrt{\tilde{J}_F/\gamma}$  and  $\sigma_z =$   
 55  $b\sqrt{J_F/\gamma}$  are, respectively, the "magnetic lengths"  
 56 along the  $x$  and  $z$  directions. From the expression  
 57 (8) a 1D equation for the function  $\chi(x)$  follows. In  
 58 the case of the equal exchange constants in FM  
 59 and AFM it takes the form

$$60 \chi(x) = -\pi - \frac{2J_S}{\pi a \sqrt{J\tilde{J}}} \quad 69$$

$$61 \times \int_{-\infty}^{+\infty} dx' K_0 \left( \sqrt{\frac{(x-x')^2}{\sigma_x^2} + \frac{b^2}{4\sigma_z^2}} \right) \quad 71$$

$$62 \sin(\chi(x')) \text{sgn}(x'). \quad 73$$

$$63 \quad (9) \quad 75$$

64 Eq. (9) can be solved by the successive approx-  
 65 imations method. For the first approximation it  
 66 gives:

$$67 \chi_1(x) = -\pi - \frac{J_S}{J^*} \sin(\varepsilon) \frac{1}{\pi} \int_{-x/\sigma_x}^{\infty} dp K_0(p), \quad (10)$$

68 where  $\varepsilon$  changes from  $-\pi$  (at  $J_S = J^*$ ) to  $-(\pi/2) \times$   
 69  $\sqrt{\gamma/J}$  (at  $J_S = J$ ). The function

$$70 I(x) = \frac{1}{\pi} \int_{-x/\sigma_x}^{\infty} dp K_0(p) \quad 85$$

71 can be estimated on the different intervals of the  
 72 coordinate  $x$  values:

$$73 I \approx \begin{cases} \sqrt{\sigma_x/|x|} \exp(x/\sigma_x) / \sqrt{\pi}, & x \ll -\sigma_x; \\ ((1 - |x|/\sigma_x) - (|x|/\sigma_x) \ln(|x|/\sigma_x)) / \pi, & -\sigma_x < x < 0; \\ ((1 + |x|/\sigma_x) - (|x|/\sigma_x) \ln(|x|/\sigma_x)) / \pi, & 0 < x < \sigma_x; \\ 1 - \sqrt{\sigma_x/x} \exp(-x/\sigma_x) / \sqrt{\pi}, & x \gg \sigma_x. \end{cases} \quad (11)$$

1 The solution of the 2D problem can be restored  
 2 by substituting the solution of the 1D Eq. (9) into  
 3 expression (8):

$$4 \varphi(x, z > 0) \approx \frac{1}{2} \frac{J_S}{J^*} \sin(\varepsilon(J_S))$$

$$5 \times \frac{1}{\pi} \int_0^\infty dp K_0(\sqrt{(p - x/\sigma_x)^2 + (z/\sigma_z)^2}).$$

(12)

6 At  $x = 0$  and  $z \gg \sigma_z$  it follows from Eqs. (12)  
 7 that  $\varphi \propto (J_S/J^*) \sin(\varepsilon(J_S)) \sqrt{z/\sigma_z} \exp(-z/\sigma_z)$ . At  
 8 large distances from the interface the system turns  
 9 to the ground state (Fig. 1).

10 In conclusion, a two-dimensional model is  
 11 presented for analytical description of the spin  
 12 structure at the FM/AFM interface with the  
 13 atomic high step. The domain wall is necessarily  
 14 associated with the step on the interface. The  
 15 energy along with the orientation of the domain  
 16 wall is dictated by the anisotropy and exchange

17 parameters of the FM, AFM and through the  
 18 interface as well as by the thickness of the layers  
 19 and geometry of the interface. The distribution of  
 20 magnetization in the entire volume of the magnet  
 21 containing the domain wall along the interface is  
 22 expressed in the terms of the magnetic and  
 23 geometrical parameters of the system. Decrease  
 24 of the nonuniformity of the magnetization dis-  
 25 tribution into the depth of the magnets is  
 26 exponential, and the width of the domain wall is  
 27 proportional to the exchange interaction in the  
 28 magnets and inversely related to the anisotropy  
 29 parameter.

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