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Determination of magnetic coupling from torque curve

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Abstract

Torque relations have been derived as a function of the coupling strength for an interfacially coupled two-layer magnetic system. The two layers are considered to be single crystal thin films characterized by uniaxial and cubic magnetocrystalline anisotropies. Both ferromagnetic and antiferromagnetic coupling cases have been studied. In the strong coupling situations, the system behaves as a single thin film with effective cubic and uniaxial anisotropy fields. The variation of the torque curve slope with the coupling strength has been studied. When the coupling is antiferromagnetic, a discontinuity in the slope value is observed at a critical coupling value where the 'spin-flop' occurs. This analysis could be used to study experimentally the nature and the strength of the magnetic coupling in a system with cubic and/or uniaxial magnetocrystalline anisotropies. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Magnetic coupling between two ferromagnetic layers interacting through an intervening non-magnetic interlayer has been a topic of great interest lately. Several techniques have been used to detect and measure the strength of the magnetic coupling. Among these experimental methods, one can cite the ferromagnetic resonance (FMR) [1–6], the magnetization curve [5,6]. The torque curves have been interpreted in different ways to measure either the magnetic anisotropy or the coupling in multilayers [7–9].

In the present work, a theoretical analysis of the torque curve as a function of coupling strength K is presented. The system modeled here consists of single crystal thin films characterized by cubic and uniaxial magnetocrystalline anisotropies. The analysis has been done for ferromagnetic and antiferromagnetic coupling cases. It will be shown that the shape as well as the slope of the torque curve depends on the magnetic coupling. This analysis could be used to study experimentally the nature and the strength of the magnetic coupling in a system with uniaxial and/or cubic magnetocrystalline anisotropies.

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2. General torque relations

The two thin film layers are denoted as A and B. They are coupled to each other through a non-magnetic layer. We consider the films with cubic and uniaxial magnetocrystalline anisotropies. This will apply for single crystal thin film layers with cubic structure. The uniaxial magnetocrystalline anisotropy has been found to exist in such thin films, this anisotropy may be due to the growth mode or it could be a stress-induced anisotropy or a surface anisotropy.

We can write the total free energy of the system per unit area as

$$E = t_{\rm A} E_{\rm A} + t_{\rm B} E_{\rm B} - K M_{\rm A} \cdot M_{\rm B},\tag{1}$$

where t_A and t_B are the thickness of layers A and B, respectively. E_A and E_B are the energies per unit volume of the individual layers A and B. These energies include, for each layer A and B, the Zeeman energy (interaction of the external magnetic field H with the magnetizations), the shape anisotropy, the cubic and uniaxial magnetocrystalline anisotropies. The last term is the inter-layer coupling energy. M_A and M_B denote the magnetizations of layers A and B, respectively, and K is a measure of the interaction. The nature of the coupling is given by the sign of K. If K is positive the energy is minimal when M_A and M_B are parallel and the coupling is ferromagnetic. If, on the other hand K is negative, then the lowest energy is achieved when M_A and M_B are antiparallel, and the coupling is said to be antiferromagnetic.

The films are taken to lie in the x-y plane, with the z-axis (the [0 0 1] direction) normal to the film plane (see Fig. 1). The external applied magnetic field **H** is rotated in the y-z plane and makes an angle α with the z-axis. The magnetization M_A is defined, in spherical coordinates, by the angles θ_A and ϕ_A ; and similarly M_B by the angles θ_B and ϕ_B . In this configuration, the total free energy of the system per unit area (Eq. (1)) can be explicitly written as

$$E = t_{A} \Biggl\{ -M_{A}H(\sin\theta_{A}\sin\varphi_{A}\sin\alpha + \cos\theta_{A}\cos\alpha) + K_{ueffA}\sin^{2}\theta_{A} + \frac{K_{1A}}{4}[\sin^{2}2\theta_{A} + \sin^{4}\theta_{A}\sin^{2}2\varphi_{A}] + \frac{K_{2A}}{16}\sin^{2}\theta_{A}\sin^{2}2\theta_{A}\sin^{2}2\varphi_{A} \Biggr\}$$



Fig. 1. Diagram of the coupled ferromagnetic layers with the crystallographic axes and the orientation of the DC magnetic field.

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$$+ t_{B} \Biggl\{ -M_{B}H(\sin\theta_{B}\sin\varphi_{B}\sin\alpha + \cos\theta_{B}\cos\alpha) + K_{ueffB}\sin^{2}\theta_{B} + \frac{K_{1B}}{4} [\sin^{2}2\theta_{B} + \sin^{4}\theta_{B}\sin^{2}2\varphi_{B}] + \frac{K_{2B}}{16}\sin^{2}\theta_{B}\sin^{2}2\theta_{B}\sin^{2}2\varphi_{B} \Biggr\} - KM_{A}M_{B} \Biggl\{ \sin\theta_{A}\sin\theta_{B}\cos(\varphi_{A} - \varphi_{B}) + \cos\theta_{A}\cos\theta_{B} \Biggr\}.$$
(2)

Here $K_{uA}(K_{uB})$ is the uniaxial anisotropy constant for layer A (layer B); K_{1A} and K_{2A} (K_{1B} and K_{2B}) are the cubic anisotropy constants for layer A (layer B). The constant K_{ueffA} (K_{ueffB}) is an effective uniaxial anisotropy constant defined as (for layer A)

$$K_{\rm ueffA} = K_{\rm uA} - 2\pi M_{\rm A}^2 \tag{3}$$

(and similarly for layer B) which takes into account the effect of the uniaxial magnetocrystalline anisotropy and of the shape anisotropy.

The first derivatives of the energy E with respect to the angles $\theta_{A,B}$ and $\phi_{A,B}$ must be equal to zero at equilibrium. The equilibrium conditions giving θ_A and θ_B will then be

$$E_{\theta A} = t_{A} \{ -M_{A}H(\cos \theta_{A} \sin \varphi_{A} \sin \alpha - \sin \theta_{A} \cos \alpha) + K_{ueffA} \sin 2\theta_{A}$$

$$+ \frac{K_{1A}}{2} [\sin 4\theta_{A} + 2 \sin^{3} \theta_{A} \cos \theta_{A} \sin^{2} 2\varphi_{A}]$$

$$+ \frac{K_{2A}}{16} [\sin^{3} 2\theta_{A} \sin^{2} 2\varphi_{A} + 2 \sin^{2} \theta_{A} \sin^{2} 2\varphi_{A} \sin 4\theta_{A}] \}$$

$$- KM_{A}M_{B} \{\cos \theta_{A} \sin \theta_{B} \cos(\varphi_{A} - \varphi_{B}) - \sin \theta_{A} \cos \theta_{B} \} = 0.$$
(4)

A similar relation holds for θ_B by changing A and B. For a relatively strong magnetic field H (the case treated in this work), the magnetizations M_A and M_B will lie in the y-z plane, i.e. $\phi_A = \phi_B = \pi/2$.

The torque per unit area of the system will then be given by

$$T = t_{\rm A} \left\{ K_{\rm ueffA} \sin 2\theta_{\rm A} + \frac{K_{\rm 1A}}{2} \sin 4\theta_{\rm A} \right\} + t_{\rm B} \left\{ K_{\rm ueffB} \sin 2\theta_{\rm B} + \frac{K_{\rm 1B}}{2} \sin 4\theta_{\rm B} \right\}.$$
(5)

Note that the torque T (Eq. (5)) does not depend explicitly on the coupling strength K. However, T does depend on K through the angles θ_A and θ_B given by Eq. (4) and its equivalent for B.

3. Torque curve slope

In the usual plot of torque, T, versus angle of applied field α , the torque curve slope is the derivative of the torque T with respect to the angle α which is different from the θ angles the magnetizations make with the *z*-axis. Torque curve slope have been used to get the anisotropy constants in single thin films [10]. In the present work, it will be shown how the slope relates to the coupling strength in a coupled layer system.

The torque T in Eq. (5) depends explicitly on θ_A and θ_B and implicitly on α . The derivative of T with respect to α will then be found by computing the partial derivatives of T with respect to θ_A and θ_B from Eq. (5) and the derivatives of θ_A and θ_B with respect to α which can be found, respectively, from Eq. (4) and the equivalent from B. Following this method, one can show that the slope is given by the expression

$$a = \frac{FX(Y+G) + GY(X+F) + KC(X+Y)(F+G)}{(X+F)(Y+G) + KC(X+F+Y+G)},$$
(6)

where

$$\begin{split} X &= t_{\rm A} M_{\rm A} H \cos(\alpha - \theta_{\rm A}), \\ F &= t_{\rm A} M_{\rm A} (H_{\rm KeffA} \cos 2\theta_{\rm A} + H_{\rm K1A} \cos 4\theta_{\rm A}), \\ C &= M_{\rm A} M_{\rm B} \cos(\theta_{\rm A} - \theta_{\rm B}). \end{split}$$

The quantities Y and G can be obtained from X and F, respectively, by changing A and B. An effective uniaxial anisotropy field and a cubic anisotropy field corresponding to K_1 have been defined for each layer, respectively, as (for layer A)

$$H_{KeffA} = \frac{2K_{uA}}{M_A} - 4\pi M_A \tag{7}$$

and

$$H_{K1A} = \frac{2K_{1A}}{M_A}.$$
(8)

(Similar relations hold for layer B.)

One can see that the slope does depend on the coupling strength K. Eq. (6) is valid for ferromagnetic as well as antiferromagnetic cases. However, recall that for ferromagnetic (antiferromagnetic) coupling, the parameter K is positive (negative); also the angles involved in the relation are not the same for the two cases.

The slope, Eq. (6), is a general formula for all angles α . For a given α value, the θ angles are found from the equilibrium conditions. In practice, one can study the slope at a particular α value in the experimental *T* versus α curve. One can choose the particular point to be, for example, $\alpha = 0$; then $\theta_A = 0$ and $\theta_B = 0$ for uncoupled and ferromagnetically coupled layers or $\theta_A = 0$ and $\theta_B = \pi$ for the strong antiferromagnetic coupling case. This particular situation ($\alpha = 0$) will be treated in Sections 5 and 6 (refer to Figs. 4 and 6).

4. Uncoupled case

When there is no coupling then K = 0. The angles θ_A and θ_B will then be given by two uncoupled equations, (Eq. (4) and its equivalent for B), which can be written (for layer A) as

$$H\sin(\alpha - \theta_{\rm A}) = \frac{1}{2}H_{\rm KeffA}\sin 2\theta_{\rm A} + \frac{1}{4}H_{\rm K1A}\sin 4\theta_{\rm A},\tag{9}$$

a similar equation holds for layer B. The torque T will then be given by Eqs. (5) and (9).

In Fig. 2, the torque per unit area, T, versus the applied DC field H angle α is plotted, for each separate layer. Two magnetically different layers have been chosen with $H_{K1A} > 0$ and $H_{K1B} < 0$. For the parameters used in the computation see the caption of Fig. 2. The total torque of a two-uncoupled-layer system will simply be the sum of the two curves (labeled A and B in Fig. 2) as is stated in Eq. (5).

The torque curve slope will then be given by Eq. (6) with K = 0; the slope can be written as

$$a = \frac{t_{A}M_{A}H\cos\left(\alpha - \theta_{A}\right)\left[H_{KeffA}\cos2\theta_{A} + H_{K1A}\cos4\theta_{A}\right]}{H\cos\left(\alpha - \theta_{A}\right) + H_{KeffA}\cos2\theta_{A} + H_{K1A}\cos4\theta_{A}} + \frac{t_{B}M_{B}H\cos\left(\alpha - \theta_{B}\right)\left[H_{KeffB}\cos2\theta_{B} + H_{K1B}\cos4\theta_{B}\right]}{H\cos\left(\alpha - \theta_{B}\right) + H_{KeffB}\cos2\theta_{B} + H_{K1B}\cos4\theta_{B}},$$
(10)



Fig. 2. Torque per unit area versus applied field angle for separate layers. Layer (A): $M_{\rm A} = 500 \text{ emu/cc}, H_{\rm KeffA} = -5.28 \text{ kOe}, K_{1\rm A} = 5 \times 10^5 \text{ erg/cc}, t_{\rm A} = 150 \text{ Å}$. Layer (B): $M_{\rm B} = 400 \text{ emu/cc}, H_{\rm KeffB} = -2 \text{ kOe}, K_{1\rm B} = -4 \times 10^5 \text{ erg/cc}, t_{\rm B} = 100 \text{ Å}$.

which is the sum of the slopes of the individual layer torque curves. For the parameters used here, the slopes, for $\alpha = 0$, are found to be equal to -3.657 and -2.666 (dyne cm/cm² deg) for layers A and B, respectively. The torque slope of the system of two uncoupled layers follows from Eq. (10) and will then be -6.323 (dyne cm/cm² deg).

5. Ferromagnetic coupling

5.1. Strong coupling case

In this case K is positive. For a very strong coupling case $K \gg 1$, the magnetizations are parallel, thus one can put $\theta_A = \theta_B = \theta$. The equilibrium condition giving angle θ can be written as

$$H\sin(\alpha - \theta) = \frac{1}{2}H_{Keff}\sin 2\theta + \frac{1}{4}H_{K1eff}\sin 4\theta,$$
(11)

where

$$H_{Keff} = \frac{t_A M_A H_{KeffA} + t_B M_B H_{KeffB}}{t_A M_A + t_B M_B}$$
(12)

and

$$H_{K1eff} = \frac{t_{A}M_{A}H_{K1A} + t_{B}M_{B}H_{K1B}}{t_{A}M_{A} + t_{B}M_{B}}$$
(13)

are defined, respectively, as an effective uniaxial magnetic anisotropy field and an effective cubic anisotropy field corresponding to K_1 of the whole system.



Fig. 3. Torque per unit area versus applied field angle for two strongly coupled layers. Ferromagnetic coupling. Parameters as for Fig. 2.

Note that if there is no uniaxial magnetocrystalline anisotropy ($K_{uA} = K_{uB} = 0$), then we may define an effective magnetization M_{eff} . From Eqs. (7) and (12), M_{eff} will be given by

$$M_{\rm eff} = \frac{t_{\rm A} M_{\rm A}^2 + t_{\rm B} M_{\rm B}^2}{t_{\rm A} M_{\rm A} + t_{\rm B} M_{\rm B}}.$$
(14)

Hence for very strong ferromagnetic coupling, the system behaves as a single thin film with an effective uniaxial magnetic anisotropy field (Eq. (12)), an effective cubic anisotropy field corresponding to K_1 (Eq. (13)). The torque will be given by Eqs. (5) and (11). In Fig. 3, the torque versus applied field angle α is plotted for two strongly ferromagnetically coupled layers. For the parameters used here (see Fig. 2), we found $H_{Keff} = -4.13$ kOe, $H_{K1eff} = 0.61$ kOe.

The torque curve slope will be obtained from Eq. (6) with K infinite; thus only the terms multiplying K will be retained. In this case one can show that the slope will be given simply by

$$a = \frac{(t_{\rm A}M_{\rm A} + t_{\rm B}M_{\rm B})H\cos(\alpha - \theta)[H_{\rm Keff}\cos 2\theta + H_{\rm K1eff}\cos 4\theta]}{H\cos(\alpha - \theta) + H_{\rm Keff}\cos 2\theta + H_{\rm K1eff}\cos 4\theta}.$$
(15)

The angle θ is given by Eq. (11); H_{Keff} and H_{K1eff} are given, respectively, by Eqs. (12) and (13). For the parameters used here, the slope, at $\alpha = 0$, for this strongly (ferromagnetic) coupled system will be equal to -6.270 (dyne cm/cm² deg).

5.2. Torque curve slope versus coupling strength

In Fig. 4, the slope at $\alpha = 0$ is plotted against the coupling strength for the ferromagnetic coupling situation. The strength of the coupling is measured conveniently by the symmetrized parameter $K' = K[M_A M_B/t_A t_B]^{1/2}$ (which has the dimension of a magnetic field). It is assumed here that the applied field is strong enough to pull both magnetizations in its direction. As one can see from Fig. 4, the slope increases slightly and in a monotonic manner with the coupling strength and levels off to the limiting value



Fig. 4. Slope of the torque curve versus coupling strength K'. Ferromagnetic coupling, $\alpha = 0$, parameters as for Fig. 2.

given by Eq. (15) for the strong coupling case. Hence the measure of the slope could lead to the coupling strength value.

6. Antiferromagnetic coupling

6.1. Strong coupling case

If the coupling is antiferromagnetic then K < 0. If the antiferromagnetic coupling is very strong, the magnetizations are antiparallel. By setting $\theta_A = \theta$, the equilibrium condition giving angle θ can be written as

$$H\sin(\alpha - \theta) = \frac{1}{2}H_{Keff}^* \sin 2\theta + \frac{1}{4}H_{K1eff}^* \sin 4\theta, \tag{16}$$

where

$$H_{Keff}^* = \frac{t_A M_A H_{KeffA} + t_B M_B H_{KeffB}}{t_A M_A - t_B M_B}$$
(17)

and

$$H_{K1eff}^{*} = \frac{t_{A}M_{A}H_{K1A} + t_{B}M_{B}H_{K1B}}{t_{A}M_{A} - t_{B}M_{B}},$$
(18)

are, respectively, an effective uniaxial magnetic anisotropy field and an effective cubic anisotropy field corresponding to K_1 of the whole system.

Hence for a very strong antiferromagnetic coupling and when the coupling is strong enough to keep the magnetizations antiparallel, the system behaves as a single thin film with, an effective uniaxial magnetic anisotropy field (Eq. (17)), an effective cubic anisotropy field corresponding to K_1 (Eq. (18)).

The torque will then be given by Eqs. (5) and (16). In Fig. 5, the torque versus applied field angle α is plotted for two strongly antiferromagnetically coupled layers. For the parameters used here (same parameters as for



Fig. 5. Torque per unit area versus applied field angle for two strongly coupled layers. Antiferromagnetic coupling. Parameters as for Fig. 2.

the ferromagnetic coupling), we found $H_{Keff}^* = -13.6$ kOe and $H_{K1eff}^* = 2$ kOe. One can see by examining Figs. 3 and 5, that the shape of the torque curve for this strong antiferromagnetic coupling case is different from that of the strong ferromagnetic one.

The slope can be evaluated for this strong antiferromagnetic coupling case by using Eq. (6). One can show that the slope in this case can be written simply as

$$a = \frac{(t_A M_A - t_B M_B) H \cos(\alpha - \theta) [H_{Keff}^* \cos 2\theta + H_{K1eff}^* \cos 4\theta]}{H \cos(\alpha - \theta) + H_{Keff}^* \cos 2\theta + H_{K1eff}^* \cos 4\theta}.$$
(19)

Here the angle θ is given by Eq. (16); H_{Keff}^* and H_{K1eff}^* are given, respectively, by Eqs. (17) and (18). For the parameters used here, the slope, at $\alpha = 0$, for this strongly (antiferromagnetic) coupled system is found to be equal to 25.375 (dyne cm/cm² deg). This slope value is quite different from that of the strong ferromagnetic case.

6.2. Intermediate coupling

If the Zeeman energy is sufficiently strong to dominate over the interfacial coupling energy, the individual film magnetic moments will be parallel to each other. If, however, the coupling is increased at constant DC field beyond a certain critical value (K_{crit}) the moment M_B is expected to 'flip' (assuming $t_A M_A > t_B M_B$) to align antiferromagnetically with M_A ; M_A will keep its previous orientation. One can also consider this in terms of critical field rather than critical coupling. Thus if the DC field is above the critical value, H_{crit} , we will have the parallel case, otherwise we will have the antiparallel situation.

The critical coupling and field can be evaluated by comparing the energies of both configurations (the magnetizations parallel and antiparallel to each other) [3]. Since K is negative for antiferromagnetic coupling, it is convenient to define a positive $K^* = -K$. For the particular case of interest ($\alpha = 0$), if H is strong enough to pull both magnetizations in its direction (case treated here), then both θ angles are equal to 0, and one can show that the critical coupling and field are related simply by [3]

$$M_{\rm A}K_{\rm crit}^* = H_{\rm crit}t_{\rm B}.$$
(20)



Fig. 6. Slope of the torque curve versus coupling strength K'^* . Antiferromagnetic coupling, $\alpha = 0$, parameters as for Fig. 2.

In this case the critical field and coupling are independent of the anisotropy constants. Thus if K^* is greater than K^*_{crit} , the magnetizations will align antiparallel to each other.

The slope is studied as a function of the coupling strength for this antiferromagnetic coupling situation. In this case K < 0, as for the ferromagnetic coupling we will measure the coupling strength by the parameter $K'^* = |K|[M_A M_B/t_A t_B]^{1/2}$. In Fig. 6, the slope versus K'^* is plotted for an applied DC field equal to 10 kOe. For this field value, the critical coupling K_{crit}^{**} is equal to 7.30 kOe (using Eq. (20)). Hence we may define two regions. For a coupling strength $K'^* < K_{\text{crit}}^*$, the magnetizations in both layers are parallel to each other and to the DC field under the strong applied field. Note however, that even though the magnetizations are pointing in the same direction, the slope is different from that of the corresponding ferromagnetic coupling case (cf. Figs. 4 and 6). When the coupling strength is increased beyond K_{crit}^{**} with the same applied field, the magnetizations will align antiferromagnetically. We see a discontinuity in the slope value (Fig. 6). Another interesting feature in the variation of the slope for this antiferromagnetic situation is the fact that it becomes infinite for a particular value of the coupling strength K'^* (around 3 kOe with the parameters used in this work), there is a vertical tangent at $\alpha = 0$. For this particular coupling value, the denominator in Eq. (6) goes to zero. At this point, no explanation could be given of the physical significance of the infinite torque slope for a certain value of the coupling strength. This phenomenon is not observed for the ferromagnetic case (see Fig. 4).

One can see therefore that the variation of the torque curve slope with coupling strength is quite different for the two cases (ferromagnetic and antiferromagnetic coupling). A study of the curve slope could lead thus to the nature of the coupling. Also for each type of coupling, the slope varies with the coupling strength (see Figs. 4 and 6). Hence a measure of the slope will give the coupling strength value.

7. Conclusions

Torque relations have been derived as a function of the coupling strength for an interfacially coupled two-layer magnetic system. The two layers are considered to be single crystal thin films characterized by uniaxial and cubic magnetocrystalline anisotropies. Both ferromagnetic and antiferromagnetic coupling cases have been studied. In the strong coupling situations, the system behaves as a single thin film with effective cubic and uniaxial anisotropy fields. The variation of the torque curve slope with the coupling strength has been studied. In the ferromagnetic coupling case, the slope increases slightly with coupling strength and levels off to a limiting value for strong coupling. On the other hand, when the coupling is antiferromagnetic, a discontinuity in the slope value is observed at a critical coupling value where the 'spin-flop' occurs. Also in the latter case, the slope can be infinite for a particular coupling value. Therefore, the variation of the torque curve slope with coupling strength is quite different for the two cases (ferromagnetic and antiferromagnetic coupling). A study of the curve slope could lead thus to the nature of the coupling and to a value of the coupling strength. This analysis could be used to study experimentally the nature and the strength of the magnetic coupling in a system with cubic and/or uniaxial magnetocrystalline anisotropies.

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