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# Temperature dependence of kinetic roughening during metal(100) homoepitaxy: transition between 'mounding' and smooth growth

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## **Abstract**

From simulations of a realistic lattice-gas model for metal(100) homoepitaxy, we analyze the temperature (*T*) dependence of the film roughness (or interface width), of the effective roughening exponent, of the local step-density, and of the persistence of the Bragg intensity oscillations. By also analyzing the dependence on *T* of the lateral mass currents of deposited atoms, we reveal a kinetic phase transition from a regime of 'mounding' at higher *T*, to a regime of 'reentrant' smooth growth at lower *T*. Application of these results for the cases of Ag, Fe, and Cu homoepitaxy is discussed. Finally, we also describe some features of the dynamics of deposited atoms that could lead to the recovery of rough growth at very low *T*. © 1999 Elsevier Science B.V. All rights reserved.

*Keywords:* Kinetic roughening; Metal(100) homoepitaxy; Mound formation; Step-edge barriers

the morphology and roughness of growing epitax- 'Poisson growth' when all thermal diffusion proial films is not only of fundamental interest, but cesses are inoperative [3]. This result extends to also of importance with regard to the control of the case where just downward interlayer transport film structure and of associated film properties [1]. is inoperative, even if intralayer terrace diffusion Two general observations have guided traditional is significant. These observations have led to the expectations regarding the *T*-dependence of homo-<br>epitaxial film of a given thickness should increase<br>epitaxial film of a given thickness should increase epitaxial growth. First, the equilibrium structure of a homoepitaxial film typically corresponds to a monotonically with decreasing temperature. Film smooth surface, as the substrate temperature is structure at higher temperatures would then reflect usually below the thermal roughening transition efficient equilibration or smoothing, and structure [2]. Consequently, the roughness of the growing at lower temperatures (where thermal diffusion is [2]. Consequently, the roughness of the growing at lower temperatures (where thermal diffusion is film provides a measure of the deviation from inhibited) would reflect kinetically limited film provides a measure of the deviation from

**1. Introduction** equilibrium. Second, analysis of random deposition for a simple-cubic (SC) crystal geometry The variation with substrate temperature (*T*) of with on-top adsorption sites reveals very rough roughening.

\* Corresponding author. Fax: <sup>+</sup>1 925-294-3231; Behavior in real systems can actually be more e-mail: mcb@io.ca.sandia.gov. complicated. In metal(111) homoepitaxy with

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threefold hollow (3FH) adsorption sites, the acti- then reentrant smooth growth occurs for lower *T* diffusion barrier) that inhibits downward transport elaborate on these issues below. at island edges. As a result, one observes rough In this paper, we provide a comprehensive char-Poisson-like growth for a broad range of *T* where acterization of the temperature dependence of film islands are large, but downward transport is growth in a model for metal(100) homoepitaxy, strongly inhibited. (Note that the above observa- as well as an analysis of the underlying atomistic tions for SC crystals apply here since crystal dynamics that controls this behavior. Specifically, or adsorption site geometry has no significant for a fixed film thickness, we examine the depeninfluence for large 2D islands.) However, for dence on *T* of the roughness (or interface width), Pt/Pt(111), 'reentrant' smooth growth was of the effective roughening exponent, of the local observed for low *T* [4,5]. This unexpected behavior step-density, and of the oscillations in the kineis apparently a consequence of a transition from matic Bragg intensity at the anti-phase condition. lowering *T*, which in turn leads to enhanced down- of the lateral mass currents of deposited atoms, ward transport, possibly due to lower step-edge we show that there is a 'kinetic phase transition' barriers at kink sites at the edges of irregular from a regime of unstable growth or 'mounding' islands [11]. for higher *T*, to a regime of smooth growth at

behavior occurs for metal(100) homoepitaxy with systems at low *T*, around and below the transition, fourfold hollow (4FH) adsorption sites [for metals can be controlled by system-specific features of the where the (100) surface does not reconstruct]. atomistic dynamics, which are not accounted for These systems are simpler than those above in that in generic modeling. In fact, a quite distinct rough no dramatic transition occurs in the shape of the growth mode could emerge at very low *T* (cf. Refs. 2D islands with decreasing *T* [12]. None the less, [27,28]). unexpected smooth quasi-layer-by-layer film growth was also observed in these systems at low *T* where terrace diffusion is inoperative [13–15]. **2. Model details and parameters for metal(100)** This behavior was explained in part by recognizing **homoepitaxy** the smoothing influence of adsorption at 4FH adsorption sites (relative to on-top sites), which We now provide a brief description of our requires four atoms in the layer beneath (compared model for metal(100) homoepitaxial growth that with just one) [16–20]. However, the other key incorporates irreversible 2D island formation in ingredient in producing smooth growth at low *T* each layer (no lateral bond-scission) [21]. The was determined to be the propensity of atoms model includes the appropriate 4FH adsorption deposited on top of isolated atoms, at step edges, site geometry and island structure for fcc $(100)$  or or on other microprotrusions to funnel downward bcc(100) metals. Specifically, the 4FH adsorption to lower 4FH adsorption sites [16–20]. For higher sites form a square grid in each layer, and the *T*, where near-square islands form in each layer, islands formed have near-square shapes with closesmooth growth is partly impeded by the presence packed  $\langle 110 \rangle$  or  $\langle \overline{1}10 \rangle$  edges aligned along the of a 'small' step-edge barrier. Recent analysis [100] and [010] principal lattice directions. The revealed that the film roughness first increases as schematic of our model in Fig. 1 indicates the *T* decreases from 'high' values, since the step-edge following key steps: barrier becomes more difficult to surmount, but 1. Atoms are deposited randomly at rate *F*, mea-

vation barrier for terrace diffusion is often low, [21–26]. The latter is due to smaller islands, a leading to the formation of large two-dimensional higher step-density, and thus enhanced downward (2D) islands within each layer [4–8]. Also, there funneling. Furthermore, the downward funneling is usually a large Ehrlich–Schwoebel or step-edge process was shown to play a crucial role in deteractivation barrier [9,10] (in addition to the terrace mining film morphology even at higher *T*. We

compact to irregular or fractal 2D islands with Furthermore, by analyzing the dependence on *T* Perhaps more surprisingly, similar 'anomalous' lower *T*. We also emphasize that behavior of actual



Fig. 1. Schematic of our model for metal(100) homoepitaxial growth without bond-scission.

atoms impinging at 4FH sites adsorb there, of additional atoms at the kink site. This feature whereas those impinging on top of isolated mimics the rapid diffusion of adatoms along atoms (or dimers or trimers), at step edges, or close-packed island edges in these systems. A on other microprotrusions funnel down to lower pair of islands does not restructure subsequent 4FH sites. A convenient discrete description of to 'collision' resulting from growth, but contindeposition dynamics is adopted, which captures ues to grow as two overlapping squares.

- sites in each layer at rate of  $h = v \exp[-E_d]$  $(k_B T)$ ] per unit time (per direction). Such adatoms also hop across descending step edges at a reduced rate  $h' = \exp[-E_{se}/(k_B T)]h$ , and there- $E_{\rm se}$  is the additional step-edge barrier, and a common attempt frequency,  $v$ , is assumed.
- 3. When two diffusing adatoms meet, they irre-
- 

sured in monolayers (ML) per unit time. Those plete edge, by the instantaneous incorporation

these features (see Appendix A). Next, we review some key quantities of interest, 2. Isolated adatoms hop to adjacent empty 4FH and their expected behavior for this model of film growth. In the submonolayer regime, the mean density of 2D islands,  $N_{\text{av}}$ , effectively saturates by a rather low value of coverage,  $\theta$ , for a typical a reduced rate  $h = \exp\left(-\frac{B_{\text{se}}}{N}\right)h$ , and there are  $\frac{B_{\text{e}}}{N}$ , and decreases at a match ingiter  $\theta$  due after funnel down to lower 4FH sites where to coalescence. At a fixed pre-coalescence  $\theta$ , one large  $h/F$ , and decreases at a much higher  $\theta$  due they are captured. (In fact, there is a finite finds the classic scaling behavior for irreversible probability that such adatoms can diffuse back island formation,  $N_{av} \sim (h/F)^{-1/3}$ , so  $N_{av}$  increases up larger {111} facets, and recross the step with decreasing *T* or increasing *F*. The mean island with decreasing  $T$  or increasing  $F$ . The mean island edge. However, if diffusion on {111} facets is separation,  $L_{av}$ , is trivially related to  $N_{av}$ , and rapid, then this process is less likely.) Here,  $E_d$  satisfies  $L_{av} = (N_{av})^{-1/2} \sim (h/F)^{1/6}$  (see Refs. [29– rapid, then this process is less likely.) Here,  $E_d$  satisfies  $L_{av} = (N_{av})^{-1/2} \sim (h/F)^{1/6}$  (see Refs. [29–is the activation barrier to terrace diffusion, 31]). In the multilayer regime, the layer coverage 31]). In the multilayer regime, the layer coverage  $L_{se}$  is the additional step edge barrier, and a distribution,  $v_j$ , for layers  $j \ge 0$ , is or primary common attempt frequency, v, is assumed. interest, where  $j=0$  denotes the substrate (so distribution,  $\theta_i$ , for layers  $j \ge 0$ , is of primary when two diffusing didentities included  $\begin{pmatrix} \cos \theta & \sin \theta \\ \cos \theta & \cos \theta \end{pmatrix}$  atoms in layer *j*, and  $\Sigma_{j\geq 0} P_j = 1$ . (Even address resolving the odge of on original sloped  $\begin{pmatrix} \cos \theta & \cos \theta \\ \cos \theta & \cos \theta \end{pmatrix}$  and  $\begin{pmatrix} \cos \theta$ =1). Then,  $P_{j\geq 0} = \theta_j - \theta_{j+1}$  gives the fraction of adatom reaching the edge of an existing island for a perfectly flat surface of height *j*, with  $P_j = 1$ <br>is inverse in large sight incorporated into that island and *P<sub>k<j</sub>*=0, the atoms in layer *j*−1 are still partly 4. During growth, individual islands maintain visible through the centers of the 4FH sites, but visible through the centers of the 4FH sites, but near-square shapes, with at most one incom- here, we treat these as 'completely' hidden.) This distribution determines the interface width, *W* (in homoepitaxy [12]. Specifically, simulation parameunits of the interlayer spacing), via [32]

$$
W^{2} = \sum_{j \ge 0} (j - j_{\text{av}})^{2} P_{j}, \text{ where } j_{\text{av}} = \sum_{j > 0} \theta_{j} = \theta.
$$
 (1)

$$
I_{BR} = \sum_{j \ge 0} (-1)^j P_j \approx 4 \cos^2[\pi \theta - \kappa \pi^3 \theta^3 + ...]
$$
  
× exp[- $\pi^2 W^2$  + ...], (2)

height) in the  $[100]$  principal lattice direction, trant smooth growth. Film snapshots obtained in which provides some assessment of the 'local slope' the simulations for various T are shown in Fig. 3. which provides some assessment of the 'local slope' of the surface in that direction.

which characterizes  $Ag/Ag(100)$  and  $Fe/Fe(100)$  However, it should be noted that even for

ters are chosen as  $E_d = 325$  meV,  $E_{se} = 25$  meV (so  $\Gamma \approx 0.077$ ,  $v=10^{12} \text{ s}^{-1}$ , and  $F=0.06 \text{ ML s}^{-1}$ , which reasonably describe  $Ag/Ag(100)$  homoepi-W quantifies the film roughness, and its increase<br>
during growth can be typically fitted by the form<br>  $W \sim \theta^{\beta}$ , where  $\beta$  is the (effective) exponent describ-<br>
ing kinetic roughening [32] (although the emer-<br>
less, ki about 200 K), the greater influence of downward funneling due to higher step-densities causes a reduction in *W* and  $\beta$  (see Fig. 2a and b). This where  $\kappa = \sum_{j \ge 0} (j - j_{av})^3 P_j / W^3$  measures the skew-<br>non-monotonic behavior, or 'reentrant' smooth was a state of the film has been noted previously [21.22]. Fig. 2c ness of the film height distribution. Below, we<br>describe how 'mounds' form during growth in the<br>presence of a step-edge barrier. These generally<br>coarsen during growth with lateral dimension satis-<br>fying,  $L_m \sim \theta^n$  [32]. I fying,  $L_m \sim \theta^n$  [32]. If a mound slope is selected, examples of oscillatory decay of the Bragg intensity<br>then  $n = \beta$ . We also consider the 'total' density of at the anti-phase condition are shown in Fig. 2d,<br>ascending pl

## *3.2. Growth beha*v*ior for large step-edge barrier*

**3. Simulation results: temperature dependence of** It is also instructive to consider the opposite **film growth** regime of 'high'  $\Gamma$ . A maximum possible value of  $\beta \approx 1/2$  is achieved if both the step-edge barrier is Below, we consider separately cases of small effectively insurmountable at the given *T* (and and large step-edge barriers,  $E_{\text{se}}$  (relative to the the refore also at lower *T*) and if the 2D islands terrace diffusion barrier.  $E_{\text{a}}$ ), and thus naturally within each layer are sufficiently large that downterrace diffusion barrier,  $E_d$ ), and thus naturally within each layer are sufficiently large that down-<br>introduce the ratio  $E_c$ ,  $E_d$ , It is instructive to ward funneling is insignificant. Behavior in the introduce the ratio  $\Gamma = E_{\rm se}/E_{\rm d}$ . It is instructive to ward funneling is insignificant. Behavior in the next that for fixed  $\Gamma$  (and fixed  $E(\omega)$  the multileventual systems case with  $\Gamma = \infty$  is shown in Fig. 4. The note that for fixed  $\Gamma$  (and fixed  $F/v$ ), the multilayer extreme case with  $\Gamma = \infty$  is shown in Fig. 4. The growth behavior of our model depends only on results in Fig. 4a and b show a monotonic increase the ratio  $E_d/T$  (or  $E_{se}/T$ ). Simulation results pre-<br>contract help was approximately contributed as the results of  $h/F$  (or *T*), as is sented below were obtained on lattices of at least expected since downward funneling diminishes 107 sites with periodic boundary conditions. with increasing 2D island size [21]. Again, the step-density increases monotonically with decreas-*3.1. Growth beha*v*ior for small step-edge barrier* ing *h*/*F* (or *T*). See Fig. 4c. Now, the persistence of the Bragg intensity oscillations decreases mono-First, we examine the case of small  $\Gamma \approx 0.1$ , tonically with increasing  $h/F$  or *T* (Fig. 4d).



Fig. 2. Temperature dependence of film growth in metal(100) homoepitaxy for small Ehrlich–Schwoebel barrier with  $\Gamma = E_{\rm se}/E_{\rm d} \approx 0.1$ . Simulation parameters are chosen to roughly match Ag/Ag(100):  $E_{\rm d} = 325$  meV,  $E_{\rm se} = 25$  meV,  $v = 10^{12}$  s<sup>-1</sup>, and  $E_{\rm e} = 0.06$  ML s<sup>-1</sup>, and  $E_{\rm e} = 0.06$  ML s<sup>-1</sup>, and  $E_{\rm e} = 0.06$  M  $F=0.06$  ML s<sup>−1</sup> [33–35]. Results for: (a) *W* at 30 ML; (b) the effective  $\beta$  determined in the range 20–30 ML (which matches the experimental values [33–35] at both 200 K and 300 K ); (c) the local slope determined from the local step-density at 30 ML; (d) antiphase Bragg intensity oscillations; data were scaled so that the maximum at  $\sim$ 1 ML has the same value for all *T*: A( $\times$ 1)-300 K;  $B(x4.4)-200$  K;  $C(x9.2)-100$  K. Curves B and C were shifted up for clarity.



Fig. 3. Snapshots of a 170a/ $\sqrt{2} \times 170a/\sqrt{2}$  region of 200 ML films obtained in simulations at  $T = 100$ , 200, and 300 K, as indicated, using  $E_d$  = 325 meV,  $E_{se}$  = 25 meV (so  $\Gamma \approx 0.08$ ),  $v = 10^{12}$  s<sup>−1</sup>, and  $F = 0.06$  ML s<sup>−1</sup>. Darker regions have a lower height.



Fig. 4. Dependence on *h*/*F* of film growth in metal(100) homoepitaxy for infinite Ehrlich–Schwoebel barrier, so  $\Gamma = E_{\rm ss}/E_{\rm d} = \infty$ .<br>Simulation graphs for (c)  $W_{\rm c}$  t 20 ML (b)  $W_{\rm c}^2$  are not for *h*/*F*<sub>a</sub> 109 Simulation results for: (a) *W* at 30 ML; (b) *W*<sup>2</sup> versus  $\theta$ , for  $h/F = 10^6 - 10^9$  as indicated (the dashed line is  $W^2 = \theta$ ); (c) the local slope determined from the local step-density at 30 ML; (d) anti-phase Bragg intensity oscillations (A:  $h/F \approx 0$ , B:  $h/F \approx 10^2$ , C:  $h/F \approx 10^4$ ). Curves B and C were shifted up for clarity.

simulations with  $F \approx 0.06 \text{ ML s}^{-1}$  predict that ing), it considered data for a narrow range of  $\theta$ ,

 $h/F \approx 10^9$  where islands are sizeable, downward roughening occurs with  $\beta \approx 0.2$  at 300 K, but with funneling is significant and the effective  $\beta$  (even a substantially higher maximum  $\beta$  of around 0.3 around  $10-20$  ML) is far below  $1/2$ . [For any fixed at 200 K (see Fig. 2b). These results (and results, range of film thickness, the effective value of  $\beta$  not shown, for the slightly higher experimental must approach 1/2 with increasing *h*/*F*. Deviations *F*#0.2 ML s−1) are in excellent agreement with in  $\beta$  from 1/2 will occur at large film thicknesses, the observations from experimental surface-sensiwhen sufficiently steep mounds develop (with tive X-ray scattering studies by Elliott et al. [36– mound height comparable to base dimension), and 38]. We also predict reentrant low values of  $\beta$ growth reflects the non-SC-geometry. The cross- below 0.3 for *T* lower than 200 K. Our simulations over thickness to this regime increases with increas- show that the 'local slope' in the [100] direction, ing  $h/F$  (see Fig. 4c).] The simulation snapshots in estimated from the total step-density, increases Fig. 5 show typical multilayer configurations for a strongly and monotonically with decreasing *T*, from about  $6^\circ$  at 300 K. It should be noted that from about  $6^\circ$  at 300 K. It should be noted that an independent X-ray scattering study by Alvarez *3.3. Application to specific systems* et al. [39] suggested that  $\beta$  decreased monotonically with decreasing *T* from 400 to 150 K in this *3.3.1. Ag*/*Ag(100)* system. This result appears inconsistent with those As noted above, the behavior shown in Fig. 2 of Elliott et al. and with our simulations. However, for a low step-edge barrier corresponds roughly to the analysis of  $\beta$  in [39] was indirect (using a  $Ag/Ag(100)$  homoepitaxial growth [33–35]. These simple heuristic rate-equation model for roughen-



100K, 200ML

- 200K, 200ML
- 300K, 200ML

Fig. 5. Snapshots of a 170a/ $\sqrt{2} \times 170a/\sqrt{2}$  region of 200 ML films obtained in simulations for  $h/F \approx 0$ , 10<sup>5</sup>, and 10<sup>8</sup> (corresponding to *T* = 100, 200, and 300 K, respectively, as indicated, if *E*<sub>d</sub> = 325 meV,  $v=10^{12}$  s<sup>−1</sup>, and *F*=0.06 ML s<sup>−1</sup>), and *E*<sub>se</sub>=∞. Darker regions have a lower height.

and it corresponded to a much lower value of *F* [22,25,26], consistent with experimental observathan [36–38]. Their more direct analysis of coars- tions [40,41]. (In contrast, such a steep slope is ening of the mean lateral mound dimension, only achieved for  $Ag/Ag(100)$  well below 295 K.)  $L_m \sim \theta^n$ , during growth (for  $\theta \le 6-8$  ML) indicated Similar success in describing growth of Fe/Fe(100) that *n* increases from about 0.15 to 0.27 for *T* was obtained in models with finite diffusion of decreasing from 300 to 200 K. This is reasonably adatoms at island edges [42,43]. consistent with the measurements of Elliott et al. and the predictions of our model, if one assumes slope selection of the mounds so that  $\beta = n$ . *3.3.3. Cu*/*Cu*(100)

 $55-65$  meV for a model with diffusion of atoms.  $(Ag)/E_d$ (Fe)]*T*\* $\approx 0.8T^*$ . Our simulations revealing a 'local slope' obtained from the total  $L_{av}$ -behavior [25,26,47], and setting  $\Gamma = \infty$ , our step-density of 14° at 295 K for  $F \approx 0.01$  ML s<sup>-1</sup> model produces far smaller  $\beta$ -values than those

The pioneering study of kinetic roughening *3.3.2. Fe*/*Fe(100)* during metal(100) homoepitaxy was actually per-From STM studies of Fe/Fe(100) growth formed for the Cu/Cu(100) system by Ernst et al. [40,41], it was estimated that  $E_d \approx 450$  meV with [44,45]. This study not only revealed mound for-<br> $\frac{40!4!}{2!}$  and that  $F = 30, 40$  meV for even motion with selected closes by also cares the  $v \approx 10^{11} \text{ s}^{-1}$ , and that  $E_{\text{se}} = 30-40 \text{ meV}$  for our mation with selected slopes, but also assessed the model (so  $\Gamma \approx 0.08$ ) [21–25], and  $E_{\text{se}} =$  T-dependence of growth. A subsequent study [46] model (so  $\Gamma \approx 0.08$ ) [21–25], and  $E_{\text{se}} =$  *T*-dependence of growth. A subsequent study [46] <br>55–65 meV for a model with diffusion of atoms supported these findings. Specific observations of along close-packed island edges controlled by a key relevance here were smoother growth at 160 K barrier of 100–125 meV [42,43]. Thus, the low (where  $\beta \approx 0.3$ ) than at 200 K (where  $\beta \approx 0.5$ ), and value of  $\Gamma$  is rather similar to that for Ag/Ag(100). a decrease in mound slopes from  $\sim$  25° at 160 K As a result, from the opening discussion in this to ~16° at 200 K (for  $F \approx 0.01$  ML s<sup>-1</sup>). Certainly, section, our model predictions for the *T*-depen- these general trends seem consistent with the predence of roughening for Fe/Fe(100) would be dictions of our model in the regime of lower *T* or similar to those for  $Ag/Ag(100)$ , for the same  $h/F$ . *h*/*F*. A simplistic interpretation of the observed More specifically, behavior observed for  $\beta \approx 1/2$  at 200 K is that the step-edge barrier is Fe/Fe(100) at a given  $T^*$  would roughly corre-<br>effectively insurmountable at this (and lower) temspond to that observed for  $Ag/Ag(100)$  at a lower peratures, assuming that the 2D islands are fairly  $T = [E_d(\text{Ag})/E_d(\text{Fe})]T^* \approx 0.8T^*$ . Our simulations large. However, choosing  $E_d \approx 0.38 \text{ eV}$  and reproduce the observed roughening, as well as  $v \approx 10^{11} \text{ s}^{-1}$  to match the measured submonolayer  $v \approx 10^{11}$  s<sup>−1</sup> to match the measured submonolayer model produces far smaller *β*-values than those observed, due to significant downward funneling [21,42,43,53–58]. More recently, it was recognized from island edges [25,26]. that the detailed behavior of film roughening and

our model. The structure of the growing tence of any counterbalancing downhill currents, Cu/Cu(100) film perhaps reflects thermodynamic  $J^{\text{down}}$ . These could result from various features of factors to a greater extent than for  $Ag/Ag(100)$  the deposition dynamics near step edges, such as or Fe/Fe(100). Indeed, the temperature for the downward funneling and 'knockout' processes transition to reversible island formation during  $[16–20]$ . Molecular dynamics (MD) studies prodeposition is much lower for Cu  $({\sim}230 \text{ K})$  than vide no evidence that 'knockout' or other transient for Ag ( $\sim$ 320 K) or Fe ( $\sim$ 500 K) [12,25,26], and processes contribute significantly to *J*<sup>down</sup> for the rates of post-deposition coarsening, and island metal(100) homoepitaxy [16–20]. Thus, below we restructuring and diffusion are higher for Cu [48– only consider downward funneling (see Fig. 6 for 50]. Thus, Cu islands could significantly restruc- a schematic). ture subsequent to collision (a feature neglected A more detailed picture of growth is based on in our model ), and perhaps diffusion of adatoms consideration of the variation of these currents up steep  $\{111\}$  facets and across step edges is with local slope,  $m$ . The currents are parallel to  $m$ , more significant. Thermodynamic effects for and, by symmetry, vanish for *m*=0. The magni-Cu/Cu(100) are likely reflected in the observed tude of  $J^{\text{up}}$  first increases with increasing  $m=|m|$ , staircase-like *T*-dependence of the selected slope but then decreases for larger *m* due to narrow during growth [46], and in the feature that terraces inhibiting lateral mass flow. The magni-'vacancy mounds' formed during ion bombard-<br>tude of  $I^{\text{down}}$  should increase roughly linearly with ment of Cu(100) select similar slopes to growth increasing step-density, and thus with *m*. Also mounds for the same *T* [51]. A study of these currents are in opposite directions. Under Cu/Cu(100) homoepitaxy at 300 K [52] revealed conditions of mounding, the magnitude of the smoother growth ( $\beta \approx 0.45$ ) and a smaller selected total current,  $\underline{J}^{\text{tot}} = \underline{J}^{\text{up}} + \underline{J}^{\text{down}}$ , will first increase slope (2.5°) than at 200 K, and was in fact from zero as *m* increases from zero, but then described in terms of thermodynamic driving decrease through zero at some  $m = m_0$ , becoming forces (e.g. coarsening due to capillarity, and a negative for large  $m$  [55–57]. (A more detailed downhill current driven by step-edge line tension). discussion of this behavior is given below.) Thus, However, our model with irreversible island forma- from an initially flat surface, the slopes of mound tion (where the downhill current has a different facets tilted in the direction of *m* will grow until origin) produces to some extent the observed selecting the stable value  $m_0$ .<br>trends, so it is difficult to ascribe the observed In the  $2+1$  dimensional systems of interest here, trends, so it is difficult to ascribe the observed behavior unambiguously to reversibility. the square substrate symmetry tends to produce

## **4. Lateral mass currents controlling film growth**

Villain [53] provided the following explanation for the occurrence of 'unstable' epitaxial growth on perfect 'singular' surfaces or substrates in the presence of a step-edge barrier. Such a barrier leads to biased reflection of diffusing adatoms from descending steps and incorporation at ascending steps. This produces a destabilizing lateral mass current, or 'Schwoebel' current, *J*up, in the uphill direction, and results in the formation Fig. 6. Schematic of the uphill 'Schwoebel' current, *<sup>J</sup>*up, and the of mounds that may coarsen during deposition downhill 'downward funneling' current, *J*DF.

This discrepancy is likely due to limitations of morphology also depends sensitively on the exis-

negative for large *m* [55–57]. (A more detailed

mounds with square pyramidal symmetry, with



facets oriented in the  $[100]$  and  $[010]$  principal just the slope of mound facets, but also their form for the dependence of  $J<sup>tot</sup>$  on  $m$  that produces selection of this specific orientation is given in Refs. [55–57], although the actual physical form must of course be more complicated. However, must of course be more complicated. However, between large and small  $M_x$ , they write [54] since we know a priori the selected [100] and [010] since we know a priori the selected [100] and [010]  $J_x^{up} \propto FS(\ell_c)^2 M_x g(\ell_c M_x)$ , with  $g(z) = 1/(1+z^2)$ .<br>orientations of the mound facets, here, it is natural to consider just the dependence of the  $[100]$  component,  $J_x$ , of the currents on the [100] component,  $m_x$ , of the slope, thus determining the selected Fortun,  $\sigma_x$ , of the slope, thus determing the selected<br> **There**, the factor  $S = 1 - \exp[-E_{se}/(k_B T)]$  rescales<br>  $\sigma_{\text{min}}$  is the slope, the direction of increasing  $\sigma_{\text{y}}$  to account for a finite step-edge barrier. This (left), for  $m_x > 0$  (<0). The discussion below is naturally cast in terms of the rescaled slope  $M_x = 2m_x$  ( $\sqrt{2}m_x$ ) for bcc (fcc) metals. For simplicity, both currents and distances will be specified in units where *a*=1.

We adopt the standard approach to quantify the lateral mass currents by simulation of deposthe lateral mass currents by simulation of depos-<br>ition on vicinal surfaces with various global slopes,<br> $m(m_x, 0)$ , which are preserved during deposition.<br> $S$  (or  $E_{\text{ee}}$ ) vanishes. However, Eq. (4) does not  $m(x,0)$ , which are preserved during deposition.<br>See Appendix B for a further discussion. Most<br>studies determine only the variation of  $J_x^{\text{tot}}$  with<br> $m_x \ge 0$ . We find it instructive to separately deter-<br>In analyzing uphi  $m_x \geq 0$ . We find it instructive to separately determine and plot  $J_x^{\text{up}} \ge 0$  and  $|J_x^{\text{down}}| \ge 0$  versus (Fig. 7), we find it most instructive to display the  $m_x \geq 0$ , the crossing point of these curves determin $m_x \ge 0$ , the crossing point of these curves determin-<br>ing the selected slope,  $m_{0x}$ . The data shown below<br>such data are shown in Fig. 8 for the case of small ing the selected slope,  $m_{0x}$ . The data shown below<br>are obtained after deposition of 20 ML for the<br>case of small step-edge barrier with  $\Gamma \approx 0.1$ . Simulation snap-<br>shots of the surface configurations for different T<br>as specifically, simulation parameters are chosen<br>as  $E_d = 450$  meV,  $E_{se} = 30$  meV (so  $\Gamma \approx 0.07$ ),  $v = 10^{11}$  s<sup>-1</sup>, and  $F = 0.7$  ML min<sup>-1</sup>, reasonably with the form (4) or with a modified form (3)

step, and those deposited further away nucleating distance,  $l_c$ , of an ascending step reaching that

into or aggregating with islands. Thus,  $\ell_c$  measures lattice directions [33–35,40,41,44–46]. Thus, not the width of the zone adjacent to ascending steps denuded of islands. Since  $l_c$  is known to scale like orientation is selected. A simple mathematical  $L_{av}$  [59], one can set  $\ell_c = AL_{av}$ , with *A* of order  $v_c = AL_{av}$ , with  $\overline{A}$  or order<br>, corresponding to the stepflow regime, the form of the current crosses over to  $J_v^{up} \sim F/M_v$ . Assuming a simple interpolation

$$
J_x^{\text{up}} \propto FS(\ell_{\text{c}})^2 M_x g(\ell_{\text{c}} M_x), \text{ with } g(z) = 1/(1+z^2). \tag{3}
$$

Here, the factor  $S = 1 - \exp[-E_{\rm se}/(k_{\rm B}T)]$  rescales slope in this direction. The direction of increasing (decreasing) film height will be identified as right form of  $J_x^{\text{up}}$  assumes that behavior is controlled by  $J_x^{\text{up}}$  assumes that behavior is controlled by a singl (left), for  $m_x > 0$  (<0). The discussion below is that other choices of  $g(z)$ , satisfying  $g(0) = 1$  and naturally cast in terms of the rescaled slope  $M_x = am_x/b$ , where '*a*' is the between lattice  $g(z) \sim 1/z^2$ , for a large *z*, are equally plausible. A *M*<sub>x</sub> = *am*<sub>x</sub>/*b*, where '*a*' is the unitional interlevent process more sophisticated analysis, by Villain and  $M_x = am_x/b$ , where 'a' is the horizontal lattice<br>spacing, and 'b' is the vertical interlayer spacing,<br>so  $M_x = 2m_x$  ( $\sqrt{2}m_x$ ) for bcc (fcc) metals. For<br>simplicity, both any of the step-flow regime for<br>simplicity, both any o

$$
J_x^{\text{up}} \propto F \ell_s \ell_c M_x / [(1 + \ell_c M_x)(1 + \ell_s M_x)], \tag{4}
$$

 $v=10^{21}$  s<sup>−</sup>, and *F*=0.7 ML min<sup>−</sup>, reasonably with the form (4), or with a modified form (3) describing Fe/Fe(100) homoepitaxy. with, say, *g*(*z*)=1/(1+*z*)<sup>2</sup>.

# *4.2. Downhill (stabilizing) current 4.1. Uphill (destabilizing) current*

For an infinite step-edge barrier, Johnson et al. For metal(100) homoepitaxy at lower *T*, the Form and the unbill Schwoebel current satis-<br>dominant contribution to the downhill current [54] argue that the uphill Schwoebel current satis-<br>fies  $J_{x}^{\text{up}} \sim F(\ell_{\text{o}})^2 M_{x}$ , for a small  $M_{x}$ . This corres-<br>panda to stame deposited on terms is a small described in Section 1.116–211 and we write ponds to atoms deposited on terraces within a described in Section 1 [16–21], and we write  $J^{\text{down}} \approx J^{\text{DF}}$ . This process must be incorporated into the modeling  $[21–26,42,43]$  to consistently repro-



Fig. 7. Simulation results for the temperature dependence of  $J_{\mu}^{\text{up}}/F$  versus  $M_x$ . Parameters were chosen to roughly match Fe/Fe(100):<br> $F_{\mu} = 450 \text{ m/s}$ ,  $F_{\mu} = 20 \text{ m/s}$ ,  $(5.07 \text{ m}) \times 10^{11} \text{ s}^{-1}$ , and  $F_{\mu$  $E_d$  = 450 meV,  $E_{se}$  = 30 meV (so  $\Gamma \approx 0.07$ ),  $v = 10^{11}$  s<sup>-1</sup>, and  $F = 0.7$  ML min<sup>-1</sup> [14,15,36–38].



Fig. 8. Data in Fig. 7 plotted as  $J_v^{up}/[FS(L_{av})^2M_v]$  versus  $L_{av}M_x$  for *T*=250, 275, 300, and 350 K, for which one has present a detailed analysis below.<br>  $L_{av} \approx 8, 11, 14,$  and 20 (in units of 'a') and  $S \approx 0.75, 0.72, 0.69$ , To this end, it is instructive to first determine

and descending steps (weighted by step height). Since this net step-density should scale like the global surface slope, one expects that

$$
J_x^{\rm DF} \approx -FC_{\rm DF} M_x,\tag{5}
$$

as recognized in early studies of funneling [16– 20]. Our simulation results confirm a near-linear dependence of  $J_x^{\text{DF}}$  on  $M_x$ . However, we should emphasize that the coefficient of proportionality,  $C_{\text{DF}}$ , is non-trivial, as it is determined by the 'locally equilibrated' film morphology, rather than by that of a perfect vicinal surface. Since the precise value of *C*<sub>DF</sub> affects slope selection, we present a detailed analysis below.

and 0.63, respectively. the relationship between  $J_x^{\text{DF}}$  and  $M_x$  for perfect vicinal surfaces (ascending from left to right for duce the observed roughening [36–38,40,41]. On  $M_x > 0$ ). Detailed analysis of our model in a vicinal surface, one expects that the magnitude Appendix C shows that  $C_{\text{DF}} = 3/8$  for a perfect a vicinal surface, one expects that the magnitude<br>of the downhill 'downward funneling' current, 'staircase' of single steps, and  $C_{\text{DF}} = (2 + k)/8$  for of the downhill 'downward funneling' current, 'staircase' of single steps, and  $C_{DF} = (2+k)/8$  for  $J_{\rm F}^{\rm DF}$ , is primarily controlled by the net step-density, height – *k* steps. This suggests that for a sufficiently *J*<sub>D</sub><sup>DF</sup>, is primarily controlled by the net step-density, height − *k* steps. This suggests that for a sufficiently i.e. the difference between densities of ascending high *T*, where the surface has predominantly singl high  $T$ , where the surface has predominantly single



Fig. 9. Snapshots of a 170a/ $\sqrt{2} \times 170a/\sqrt{2}$  region of 0.5 and 50 ML films obtained in simulations at 200 K (*h*/*F*  $\approx$  10<sup>2</sup>) and 300 K  $(h/F \approx 10^6)$ , as indicated, for a perfect vicinal surface, using parameters as in Fig.7. Darker regions have lower height.

steps, one should find that  $C_{DF} \approx 3/8$ , and also that  $C_{\text{DF}}$  should increase for a lower *T* where a greater number of multiple steps likely occur due to higher local slopes. Indeed, the simulation data for our model shown in Fig. 10 for a small stepedge barrier with  $\Gamma \approx 0.1$  are entirely consistent with this proposed trend.

Of course, a picture of the growing surface as a staircase of equal height steps is overly simplistic. Consider the evolution of a surface during deposition from an initial perfect staircase of singlesteps, with a small  $M_x > 0$ . Initially, isolated islands are formed on the broad terraces, but this has little effect on  $J_x^{DF}$  since there are equal contribu-<br>tions to the current to the left and right from tions to the current to the left and right from Fig. 10. Simulation results for the temperature dependence of funneling off left and right island edges, respec-<br>tively. As growth continues, islands merge with  $\frac{J_{\text{av}}^$ tively. As growth continues, islands merge with give  $C_{DF} \approx 0.491, 0.467, 0.440, 0.41$  ascending steps, which effectively replaces straight 275, 300, and 350 K, respectively. ascending steps, which effectively replaces straight



steps with meandering steps (cf. Fig. 9), and should and there is a kinetic phase transition from reaching descending steps effectively replace a selected slope,  $M_{0x}$ , also decreasingle-step-up configuration with a paired double-<br>decreases toward this transition. single-step-up configuration with a paired doublestep-up and single-step-down configuration, which The observed variation of  $M_{0x}$  with *T* does not increases  $|J_x^{DF}|$  (see Appendix C). correspond to the previously described behavior

must be sufficiently strong that  $J_x^{\text{tot}} = J_x^{\text{up}} +$  the net step-density, in contrast to the 'local slope'<br> $J_y^{\text{down}} > 0$  for a small  $M_x$ . Let  $J_y^{\text{DF}} = -FC_{\text{DE}}M_x$ , that is determined from the total step-density.  $J_{\text{av}}^{\text{down}} > 0$  for a small  $M_x$ . Let  $J_x^{\text{DF}} = -FC_{\text{DF}}M_x$ , that is determined from the total step-density. and assume that  $J_x^{up} \approx AFS(L_{av})^2 M_x$  for small However, it is plausible that the 'local slope' and  $M_x$ , with A of order unity. Then one requires that the selected slopes would be similar in magnitude  $M_x$ , with *A* of order unity. Then one requires that the selected slopes would be similar in magnitude  $\frac{M_x}{M}$  and  $\frac{M}{M}$  and  $\frac{$  $\partial J_x^{\text{up}} / \partial M_x \approx AFS(L_{\text{av}})^2$  exceeds  $|\partial J_x^{\text{DF}} / \partial M_x$  $FC<sub>DF</sub>$ , as is expected for a moderate and higher *T* our simulations mimicking Fe/Fe(100) growth at (where  $L<sub>av</sub>$  is large). Then, as noted in Section 4, 300 K, both slope estimates agreeing with experi-(where  $L_{av}$  is large). Then, as noted in Section 4,  $300 \text{ K}$ , both slope estimates agreeing with experitive mound facets oriented in the [100] direction mental observations. A similar consistency is the mound facets oriented in the [100] direction select a slope,  $M_{0x}$ , which satisfies the condition  $J_x^{\text{up}} = |J_x^{\text{DP}}|$ . For a large selected slope corresponding for the selected slope are not available. We have to the step-flow regime  $(\ell_{c} M_{x})$ (4) of Villain et al. [60,61] for  $J_x^{\text{up}}$  behavior for multilayer Cu/Cu(100). Finally, we  $\approx BF_s/(1+\ell_sM_x)$ , with *B* of order unity, implies note that the selected slope,  $M_{0x}$ , from simulation that *M*  $\approx$   $[61/\ell^2 +$  $/(1+\ell_s M_x)$ , with *B* of order unity, implies note that the selected slope,  $M_{0x}$ <br>*M*<sub>0x</sub>  $(1/\ell^2 + 4R/C)^{-1/2}$ ,  $1/\ell$ ,  $1/2$ , which is results for the surrents correspondent  $\sum_{s=1}^{\infty} \frac{D \cdot \sum_{s=1}^{N} (1 + v_s H_x)}{v_s}$ , which is that the selected slope,  $H_{0x}$ , from simulation that  $M_{0x} \approx [(1/\ell_s^2 + 4B/C_{\text{DF}})^{1/2} - 1/\ell_s]/2$ , which is results for the currents corresponds to that for  $\frac{1}{\sqrt{6}}$  expected to increase with decreasing *T* (corre-<br>expected to increase with decreasing *T* (corresponding to increasing  $\ell_s$ ). The same trend is found using  $J_x^{\text{up}} \propto \text{FS}/M_x$  from Eq. (3), or other modified expressions for  $J_x^{\text{up}}$  [62]. However, corrections to  $J_x^{up}$  for smaller slopes could produce the opposite trend. Thus, for a precise and general analysis of **6. 'Anomalous' behavior for low temperature** the behavior of  $M_{0x}$ , it is appropriate to utilize the<br>
simulation results for the survants from Section 4. simulation results for the currents from Section 4. *6.1. Long-range lateral correlations at low T*

In Fig. 11, we show simulation results for the behavior of  $J_x^{\text{up}}$  and  $|J_x^{\text{DF}}|$  versus  $M_x$ , for a small step-edge barrier with  $\Gamma \approx 0.1$  and for a broad one does not expect significant island formation, range of *T*. For a higher *T*, the scenario indicated so submonolayer lateral spatial correlations in the above for mound formation and slope selection adlayer should be of a short range. However, highapplies. However, the selected slope,  $M_{0x}$ , varies only weakly (decreasing slightly) over a broad large lateral correlation lengths of  $\sim$  10a persist to range of *T*. As the temperature decreases further temperatures as low as 80 K [63,64]! One possible and thermal diffusion is strongly inhibited, one source of such a correlation derives from the very finds that  $J_x^{up}$  is reduced significantly, and the low activation barriers that exist for both thermal value of  $\partial J_x^{up}/\partial M_x$  at  $M_x = 0$  decreases below diffusion along close packed step edges, and for value of  $\partial J_x^{up}/\partial M_x$  at  $M_x = 0$  decreases below diffusion along close packed step edges, and for  $\partial J_x^{up}/\partial M_x$  at this point, the unbill disconsulus discontract provessing a point  $|\partial J_x^{\text{DF}}/\partial M_x| \approx FC_{\text{DF}}$ . At this point, the uphill Schwoebel current becomes too small to counterbalance the downhill downward funneling current, could allow significant restructuring or 'clumping'

not significantly affect  $J_{x}^{\text{DF}}$ . However, any islands "mounding' to 'smooth growth' with  $\beta \approx 0$ . The selected slope,  $M_{0x}$ , also decreases to zero as *T* 

correspond to the previously described behavior of the 'local slope' obtained from the local stepdensity. The latter increases monotonically with **5. Mounding, slope selection, and the kinetic phase** decreasing *T* and is non-zero for smooth growth **transition** (below the transition) with  $\beta \approx 0$ . This difference should be expected since the selected 'global' slope For the formation of mounds, the uphill current of the sides of large mounds is determined from for *T* well above the transition. This is found in expected for  $Ag/Ag(100)$ , where experimental data already noted that our model does not describe experimental behavior for sufficiently thick films (cf. Ref.  $[52]$ ).

For low *T*, where terrace diffusion is inoperative, resolution diffraction studies have revealed that diagonally adjacent adatoms to move into a neighboring configuration  $[25,26,65]$ . Together, these



Fig. 11. Temperature dependence of the variation of the uphill and downhill currents with slope (data from Figs. 7 and 9). This sequence illustrates the kinetic phase transition as *T* decreases and the curves for the two currents 'uncross'.

of the adlayer deposited at a low *T* where most still be quite rapid due to a much lower activation atoms land near to other adatoms (M. Breeman, barrier. Thus, although atoms deposited on such pers. commun.; see also [66]). Another possible microfacets may be thermally accommodated source of 'clumping' at low *T* in metal(100) homo- before reaching 4FH sites at the bottom, one might epitaxial systems derives from the feature that expect that they will quickly diffuse randomly deposited atoms have a significant excess kinetic around the microfacets until they become trapped energy upon impact. Whereas one does not find a at the 4FH sites at the bottom which provide significant 'transient mobility' of isolated deposited 'sinks' for the diffusing adatoms [22]. This type of atoms [16–20], there may be some 'transient thermal motion might be viewed as constituting a clumping' of atoms deposited near to other ad- downhill Schwoebel current that would effectively atoms [25,26,67]. Finally, we note that these low recover the same growth behavior as for 'perfect' *T* clumping processes would likely provide an downward funneling [22]. additional contribution to the uphill current, not However, the steep local microprotrusions, typiselection at a lower *T* and the kinetic phase complicated local geometries. Some of these will transition. likely trap deposited adatoms at epitaxial non-4FH

quent more detailed analyses [68], it was noted slopes'. However, for metal(100) homoepitaxial least partly compensating for this breakdown. In isolated atoms on  $\{100\}$  terraces is typically negli- be higher than the diffusion barrier on  $\{111\}$ 

included in our modeling. This would modify slope cally present in low *T* growth, present a variety of sites more effectively than do the 3FH sites on the *6.2. Breakdown of downward funneling* sides of simple {111} microfacets. We can categorize such 'trap sites' according to the number  $(p)$ In the initial MD studies demonstrating the of supporting atoms in the layer beneath, and the existence of the downward funneling process in number (*q*) of in-layer nearest-neighbor atoms, metal(100) homoepitaxy [16–20] and in subse-<br>guant maps detailed analyses [681, it was noted sites are  $S_0^{\text{q}}$  sites, and  $2\text{EH}$  sites an (111) migrafies sites are  $S_4^q$  sites, and 3FH sites on  $\{111\}$  microfacthat depositing atoms can be caught at threefold ets are  $S_2^1$  (and  $S_1^2$ ) sites. [ $S_2^1$  sites are not epitaxial<br>hallow (2EU) sites an the sides of sufficiently large with account to the graving for a her smutch l. hollow (3FH) sites on the sides of sufficiently large with respect to the growing fcc or bcc crystal.] We {111} microfacets, rather than funneling to the expect that the  $S_3^q$  sites could play a particularly bottom. Such microstructures might be more prev- important role as 'traps', and such sites should alent at lower *T* where there are larger 'local have (statistically) significant populations at least for  $q=0$  and  $q=1$ . Although the S<sub>3</sub><sup>3</sup> sites may not growth, we expect a further important effect at  $\qquad$  trap that effectively, certainly  $S_3^1$  sites will be much least neathly compared in  $S_3$  and  $S_4$  and  $S_5$  and  $S_6$  and  $S_7$  and  $S_7$  and  $S_8$  and  $S_7$  and more likely to trap than  $3FH S<sub>2</sub><sup>1</sup>$  sites (see Fig. 12). the lower *T* regime around 100 K, diffusion of The barrier to escape from  $S_3^1$  sites will no doubt gible, but diffusion on {111} microfacets may well facets, so adatoms trapped at such sites will have



Fig. 12. Schematic of key trap sites,  $S_1^2$ ,  $S_2^1$ ,  $S_3^1$ ,  $S_3^1$  and  $S_3^2$ , that 1g. 12. Schemate of Rey trap sites, 51, 52, 53, 53 and 53, that<br>induce a breakdown of downward funneling deposition **7. Conclusions** dynamics, and enhance overhang and defect formation.

Trapping at these sites can thus significantly influ- a realistic model of metal(100) homoepitaxy. In ence the film growth mode and morphology, as particular, we identified a kinetic phase transition discussed below. from a regime of 'mounding' at higher *T*, to one

a non-4FH site will not be able to escape via any (asymptotic) exponents [32] has limited practical thermal diffusion process (although the possibility applicability in the regime of interest for these of 'knockdown' by subsequently deposited atoms experiments. exists). Thus, one should expect a breakdown of In closing, we comment briefly on other related the predictions of growth models incorporating work, and on natural extensions to this study. simple downward funneling. It is clear that once Recently, it was observed that the presence of non-4FH epitaxial sites can be populated, the long-range attractions between diffusing adatoms possibility exists for the formation of overhangs, and step-edges induces a destabilizing uphill curand even enclosed voids or defects. Indeed, the rent. This leads to mounding, even in the absence spontaneous creation of voids has been observed of an Ehrlich–Schwoebel barrier [73]. It is not yet in recent MD studies of low-*T* metal(100) homo- clear whether this effect is significant in metal(100) epitaxy [27,28]. No doubt, a detailed analysis of homoepitaxy. Another study proposed a transition the mechanistic genesis of their formation would from 'mounding' to smooth growth as *T* increases reveal the key role of trapping at  $S_3^q$  sites. The reveal the key role of trapping at  $S_3^q$  sites. The above some 'high' critical value (which is distinct other key observation of these MD studies was from the low-T transition discussed above) [62]. rough film growth. Certainly, precise characteriza- However, the underlying analysis utilized an tion or prediction of roughening would depend on expression for  $J^{\text{up}}$  based on perfect vicinal surfaces the details of the breakdown of funneling, and of in the step-flow regime, so its accuracy is unclear the formation of overhangs and voids. However, near a transition where the selected slope vanishes some generic insight into the roughening of films and island nucleation becomes significant. Future incorporating bulk defects is available. It is well studies should systematically explore the influence

known that in simplistic 'epitaxial growth' models with bulk defects, such as 'ballistic deposition' [69] and 'random sequential adsorption' [70,71], roughening is described by non-linear Kardar– Parisi-Zhang (KPZ) evolution. Here, one has  $W \sim \theta^{1/4}$  (in 2+1 dimensions), if enclosed voids are ignored in the determination of *W* [32]. The origin of the KPZ non-linearity is simply that for deposition on a vicinal substrate, the volume density of bulk defects formed depends on the substrate tilt. Since this is expected to be a feature of more realistic models (cf. Ref. [72]), it is reasonable to expect KPZ-type roughening for low *T* deposition in metal(100) homoepitaxy.

We have provided a comprehensive analysis of more difficulty in reaching lower 4FH sites. the temperature (*T*) dependence of roughening for of reentrant 'smooth growth' at lower *T*. All *6.3. Multilayer growth at*  $T \approx 0$  *K* quantities of interest vary strongly with *T* in the mounding regime, including the effective exponent At  $T \approx 0$  K, an atom that is accommodated at for roughening. Thus, the concept of universal

from the low- $T$  transition discussed above) [62].

of reversibility in submonolayer island formation on subsequent multilayer growth. Reversibility results in enhanced depletion of nearby 2D island pairs in the submonolayer regime [29,74]. This leads to greater 'ordering' of 2D islands, and thus of the multilayer mounds, as well as enhanced splitting of diffraction profiles [21]. Finally, for a reliable analysis of the unusual features of growth at low *T*, it would be appropriate to introduce refined system-specific models accounting for both intralayer 'clumping' of nearby adatoms, and trapping of deposited atoms at non-4FH sites.

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Office of Basic Energy Sciences DOE by Sandia Corporation, a Lockheed Martin Company, under Contract No. DE-AC04- **Appendix B: Simulation of slope dependence of** 94AL85000. **mass currents**

film for fcc or bcc metal(100) homoepitaxy reveals currents determined after incremental deposition two interpenetrating subarrays of vertical columns on a perfect vicinal surface display a qualitatively of atoms, labeled '+' and '−' in Fig. 13. Atoms reasonable *m*-dependence [42,43,54], more proat the top of one set of columns are in even layers, longed deposition is appropriate. For larger and those in the other are in odd layers. For slopes,  $m_x \ge m_{0x}$ , equilibration is presumably rapid,  $m_x$  and  $m_y$  and  $m_z$  and  $m_z$  and  $m_z$  and the habitation in this noting is growth with no overhangs or internal vacancies, and we emphasize that behavior in this regime is the top atoms in adjacent columns of different actually sufficient to determine  $m_{0x}$ . However, for subarrays have a height difference of unity. This geometry is equivalent to that of the so-called single-step model for film growth [75]. As in previous studies [16–26], we naturally replace choice of finite deposition time or finite system deposition with continuous lateral coordinates size  $(\gg L_{av})$ . In the simulations, we used perfect with deposition on top of 'discrete' columns chosen vicinal surfaces of single-steps where 20 ML were



Fig. 13. Bird's-eye view of the geometry of a growing surface **Acknowledgements** during metal(100) homoepitaxy. Subarrays of vertical columns are indicated by  $+$  and  $-$ .

**Appendix A: Discrete treatment of deposition** For elucidation of the morphology of growing **dynamics** deposited atoms for a 'locally equilibrated' growing surface is relevant (rather than behavior for a A bird's-eye view of the surface of a growing perfect vicinal surface) (see Fig. 9). Thus, whereas  $\langle m_{0x}$ , an infinite growing surface will facet into regions of selected slope,  $\pm m_{0x}$ , and so, strictly, *J*'s should be measured for a judicious vicinal surfaces of single-steps where 20 ML were



Fig. 14. Top and side views of a periodic vicinal staircase of single-steps (up from left to right). Here,  $k=1$ ,  $L=4$ , and  $\lambda=9/2$ , and  $p_A = 1/2, d_A = -1/2, p_B = 1, d_B = -1/2.$ 

deposited. The currents were then monitored for on top of the two subarrays of vertical columns

# Appendix C: Analysis of the downward funneling

is measured in the [100] the nopping in-plane  $(a_i$  is measured in units of direction. We determine the mean lateral displace-<br>ment of deposited atoms  $\langle l \rangle$  and thus the lateral displacement of atoms deposited within a ment of deposited atoms,  $\langle l_{\text{DF}} \rangle$ , and thus the lateral displacement of atoms deposited within a lateral displacement of atoms deposited within a lateral lateral flux,  $J_x^{DF} = F(\ell_{DF})$ , due to the downward period ( $\lambda$ a) is  $\ell_{TOT} = \sum_i p_i d_i$ , and the mean lateral function depending Function depending Function depending  $\ell_{TOT}$ ) =  $\ell_{TOT}$  ( $\ell$ ) funneling deposition dynamics. Furthermore, we note the consistency between continuum and discrete treatments of deposition [16–20]. In the *k* atoms and terraces with *L* fully exposed atoms, former, atoms impinge on the surface with con-<br>scending from left to right (see Fig. 14). Here, former, atoms impinge on the surface with continuous lateral positions chosen randomly well the lateral periodicity is  $\lambda = L + k/2$ , in units of above the surface. One determines the lateral dis-<br>  $a^2$ , and the global slope is  $M_x = k/\lambda$ . In this case,<br>
placement for each position, and obtains a mean<br>
one finds a mean lateral displacement to the left placement for each position, and obtains a mean displacement after integrating over one period of the vicinal surface (and then normalizing). In the latter, atoms deposit randomly at discrete locations

an additional 0.2 ML. of atoms composing the film. Now, one determines the lateral displacement for each discrete position, and obtains a mean after summing over one period. Specifically, we let  $p_i$  denote the probability of **current current current funneling** down for an atom deposited on top of site i, and  $d_i$  the extra lateral displacement to the i We consider atoms depositing on two classes of lower 4FH site on funneling down, compared with the hopping in-plane  $(d_i$  is measured in units of period ( $\lambda$ a) is  $\ell_{\text{TOT}} = \sum_i p_i d$ displacement is  $\langle l_{\text{DF}} \rangle = l_{\text{TO}}$ <br>Class 1: A perfect stairs

Class 1: A perfect staircase with steps of height  $k$  atoms and terraces with  $L$  fully exposed atoms, of  $\langle \ell_{DF} \rangle = k(k+2)/(8\lambda)$ , in units of 'a', so  $|J_x^{DF}|$  $= F(k+2)M_{x/8}$ , and  $C_{DF} = (k+2)/8$ , as stated in the text. We note that our result  $C_{\text{DF}}=3/8$  for



Fig. 15. Top and side views of a periodic vicinal staircase of pairs of double-step up and single-step down, from left to right. Here,  $k=1, L=9$ , and  $\lambda=19/2$ , and  $p_A=1/2, d_A=+1/2, p_B=1, d_B=+1/2, p_C=1/2, d_C=-1, p_D=1, d_D=-1, p_E=1, d_E=-1/2$ .



Fig. 16. Snapshots of  $170a/\sqrt{2} \times 170a/\sqrt{2}$  regions of periodic vicinal staircases with (a) single-steps up, and (b) pairs of double-step up and single-step down. Darker regions have a lower height. (c) Corresponding simulation results for  $|J_x^{\text{DF}}|/F$  versus  $M_x$ . The slopes give  $C_{\text{DF}} \approx 0.377$  (Class 1) and  $C_{\text{DF}} \approx 0.629$  (Class 2).

 $k=1$  (Fig. 14) appears inconsistent with a calcu-<br>lation in Refs. [42,43], but this is only because of height  $k+1$  atoms up, and height 1 atom lation in Refs. [42,43], but this is only because of height  $k+1$  atoms up, and height 1 atom part of  $J_r^{DF}$  was assigned to (i.e. subtracted from) down, and lateral periodicity  $\lambda = L + k/2$  (see part of  $J_x^{DF}$  was assigned to (i.e. subtracted from)  $J_x^{up}$  in that analysis.

Fig. 15). Here, the global slope is  $M_x = k/\lambda$ , as in

Class 1, but now one finds a mean lateral displace-<br>ment to the left of  $\ell \rightarrow -k(k+4)/(8.3)$ , so  $LPF_1$  [26] J.W. Evans, M.C. Bartelt, in: M.C. Tringides (Ed.), Surhas  $C_{\text{DF}} = 5/8$  for a paired double-step up and  $\frac{V_{\text{Ork}}}{V_{\text{Ork}}}} = 197$ , p. 197.<br> *Single-step* down staircase (Fig. 15), versus [27] C.L. Kelchner, A.E. DePristo, J. Vac. Sci. Technol. A 14  $single-step$  down staircase (Fig.15), versus  $C_{\text{DF}} = 3/8$  for a perfect single-step vicinal staircase (1996) 1633.<br>(Fig. 14) with the same slope Simulations dupli- [28] C.L. Kelchner, A.E. DePristo, Surf. Sci. 393 (1997) 72. (Fig. 14) with the same slope. Simulations dupli-<br>cate these exact results (see Fig. 16).<br>[29] J.A. Venables, Phil. Mag. 27 (1973) 697.<br>[30] M.C. Bartelt, J.W. Evans, Surf. Sci. 298 (1993) 421.

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