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## Spin Waves in a Bilayer with Biquadratic Interlayer Coupling

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Linear spin waves in layered structures consisting of two ferromagnetic films coupled at the interface by a biquadratic exchange interaction are analysed theoretically within the localized spin model and in the low-temperature limit. Magnetizations of both films are assumed to be parallel to the film plane, but perpendicular one to another. The limit of two semi-infinite ferromagnets is also considered. In both cases interfacial modes have been found.

### 1. Introduction

Indirect exchange interaction between magnetic films separated by a nonmagnetic layer has usually two components, bilinear and biquadratic ones [1]. The bilinear term originates from interference of electron waves reflected from interfaces between the magnetic and nonmagnetic layers. The origin of the biquadratic term is more complex and several different contributions have been proposed. Those include an intrinsic term [2], which is of the same physical origin as then bilinear term mentioned above, and some extrinsic terms which are either due to interfacial roughness [3] or due to loose spins in the nonmagnetic material separating the ferromagnetic films [4].

The biquadratic coupling in real systems usually favours perpendicular alignment of the film magnetizations (negative value of the corresponding coupling parameter) [1], while the bilinear one favours either parallel or antiparallel configuration. Interplay of both terms can lead to a noncollinear orientation of the film magnetizations and to a complex phase diagram in an external magnetic field [5 to 7]. In some cases the biquadratic coupling is stronger than the bilinear one, leading to perpendicular orientation of the film magnetizations. This was observed experimentally e.g. in Fe/Cr/Fe [1] or Fe/Al/Fe [8] trilayers.

Spin waves in layered magnetic structures were usually analysed theoretically for bilinear interlayer coupling and for collinear (parallel and antiparallel) orientations of the film magnetizations [9 to 18]. Also experimental analyses were restricted mostly to the parallel and antiparallel configurations. This is due to the fact that the spin wave spectrum is then simpler and experimental data are easier for interpretation [10]. The discovery of biquadratic interlayer coupling renewed the interest in spin-wave analysis, particularly for noncollinear configurations [6, 7]. Such an analysis can give important information about the coupling parameters.

Spin waves in a system composed of two atomic planes coupled by bilinear and biquadratic exchange interactions were considered in [6], where the authors analysed in detail

the phase diagram of the system in an external magnetic field as well as the corresponding spin-wave spectrum. Here we consider the case where the magnetic films have arbitrary thickness. However, the analysis is restricted to perpendicular alignment of the film magnetizations. Such a configuration can occur when the biquadratic coupling is strong enough to overcome the bilinear term (provided its sign favours perpendicular alignment). The interplay of exchange terms and magnetic anisotropy can then lead to perpendicular alignment of the magnetizations in the ground state configuration. In the following we simplify the situation by assuming no bilinear interlayer coupling.

The system and model Hamiltonian are described in Section 2. Equations for Green's functions and spin wave frequencies are derived in Section 3. The limit of two semi-infinite ferromagnets coupled at the interface by a biquadratic interaction is considered in Section 4. In Section 5 we present some numerical results for two exchange-coupled thin films. Concluding remarks are given in Section 6.

## 2. Description of the Model

We consider two films consisting respectively of  $N_1$  and  $N_2$  (100) atomic planes of a simple cubic lattice. The lattice sites are occupied by localized magnetic moments which are coupled ferromagnetically within each film. Both films are coupled at the interface by a biquadratic exchange interaction. The biquadratic coupling is assumed to be sufficiently strong to neglect the bilinear term. To stabilize the film magnetization we include a single-ion cubic anisotropy in the ferromagnetic films. Thus, the Hamiltonian corresponding to the system can be written in the form

$$\begin{aligned}
 H = & -\frac{1}{2} \sum_l \sum_{\mathbf{q}} \sum_{\delta_{\parallel}} J_l \hat{\mathbf{S}}_{l\mathbf{q}} \cdot \hat{\mathbf{S}}_{l\mathbf{q}+\delta_{\parallel}} - \sum'_l \sum_{\mathbf{q}} J_{l,l+1} \hat{\mathbf{S}}_{l\mathbf{q}} \cdot \hat{\mathbf{S}}_{l+1\mathbf{q}} \\
 & - \sum_{\mathbf{q}} J_b (\hat{\mathbf{S}}_{N_1\mathbf{q}} \cdot \hat{\mathbf{S}}_{N_1+1\mathbf{q}})^2 + \sum_l \sum_{\mathbf{q}} K_l (\hat{S}_{l\mathbf{q}}^{x2} \hat{S}_{l\mathbf{q}}^{y2} + \hat{S}_{l\mathbf{q}}^{x2} \hat{S}_{l\mathbf{q}}^{z2} + \hat{S}_{l\mathbf{q}}^{y2} \hat{S}_{l\mathbf{q}}^{z2}). \quad (1)
 \end{aligned}$$

Here  $J_l$  is the exchange integral for nearest neighbours (NNs) within the  $l$ -th atomic plane ( $l = 1, \dots, N_1 + N_2 \equiv N$ ), and  $J_{l,l+1}$  is the exchange integral for NNs lying in two adjacent ( $l$ -th and  $(l+1)$ -st) atomic planes. The prime at the sum over  $l$  in the second term means that now  $l$  ranges from  $l = 1$  to  $N - 1$ . The vector  $\mathbf{q}$  is the position vector within the atomic planes, whereas  $J_b$  is the biquadratic exchange integral between the planes corresponding to  $l = N_1$  and  $N_1 + 1$  (interlayer biquadratic coupling). Finally,  $K_l$  is the anisotropy constant for the  $l$ -th atomic plane. Translational invariance in the film plane is assumed in Eq. (1), while no such a symmetry exists along the normal direction. All the intra-layer exchange integrals are assumed ferromagnetic,  $J_{l,l+1} > 0$  (for  $l \neq N_1$ ) and  $J_l > 0$  (for all  $l$ ), while the interlayer bilinear term vanishes,  $J_{N_1, N_1+1} = 0$ . The biquadratic constant  $J_b$  is negative,  $J_b < 0$ , so the biquadratic coupling favours configuration with perpendicular alignment of the film magnetizations. Finally, the anisotropy constants  $K_l$  are assumed positive, so the magnetic easy axes are along the main crystallographic directions. The above assumptions lead to the magnetic ground state, in which the magnetizations of both layers are parallel to the film plane, but perpendicular to one another.

Since our main objective is the analysis of linear excitations in the low-temperature limit, we perform the Hollstein-Primakoff transformation of the Hamiltonian (1), fol-

lowed by the in-plane Fourier transformation. In the harmonic approximation the Hamiltonian (1) takes then the form

$$\begin{aligned}
H = & \sum_{\mathbf{k}} \sum_l [(4SJ_l[1 - \gamma_{\parallel}(\mathbf{k})] + J_{l,l+1} + J_{l,l-1} + g\mu_B H_l^{\text{an}}) \hat{A}_{l\mathbf{k}}^+ \hat{A}_{l\mathbf{k}} \\
& - SJ_{l,l+1} (\hat{A}_{l\mathbf{k}}^+ \hat{A}_{l+1\mathbf{k}} + \hat{A}_{l\mathbf{k}} \hat{A}_{l+1\mathbf{k}}^+)] \\
& - \frac{J_b S^3}{2} \sum_{\mathbf{k}} [\hat{A}_{N_1\mathbf{k}}^+ \hat{A}_{N_1-\mathbf{k}}^+ + 2\hat{A}_{N_1\mathbf{k}}^+ \hat{A}_{N_1\mathbf{k}} + \hat{A}_{N_1\mathbf{k}} \hat{A}_{N_1-\mathbf{k}} \\
& - \hat{A}_{N_1+1\mathbf{k}}^+ \hat{A}_{N_1+1-\mathbf{k}}^+ + 2\hat{A}_{N_1+1\mathbf{k}}^+ \hat{A}_{N_1+1\mathbf{k}} - \hat{A}_{N_1+1\mathbf{k}} \hat{A}_{N_1+1-\mathbf{k}} \\
& - 2i(\hat{A}_{N_1\mathbf{k}}^+ \hat{A}_{N_1+1\mathbf{k}} + \hat{A}_{N_1\mathbf{k}} \hat{A}_{N_1+1-\mathbf{k}} - \hat{A}_{N_1\mathbf{k}}^+ \hat{A}_{N_1+1-\mathbf{k}}^+ - \hat{A}_{N_1\mathbf{k}} \hat{A}_{N_1+1\mathbf{k}}^+)], \quad (2)
\end{aligned}$$

where a constant term has been omitted. Here  $g$  and  $S$  are respectively the Landé factor and spin number, and the operators  $\hat{A}_{l\mathbf{k}}^+$  and  $\hat{A}_{l\mathbf{k}}$  are the in-plane Fourier transforms of the spin deviation operators.  $H_l^{\text{an}}$  is the effective anisotropy field corresponding to the single-ion anisotropy assumed here, while  $\gamma_{\parallel}(\mathbf{k})$  is defined as

$$\gamma_{\parallel}(\mathbf{k}) = \frac{1}{2} (\cos k_x a + \cos k_z a), \quad (3)$$

with  $a$  and  $\mathbf{k}$  being, respectively, the lattice parameter and in-plane wavevector. The coordinate axes are along the main crystallographic directions, with the  $y$ -axis being normal to the film plane.

### 3. Green's Functions and Spin Wave Modes

Let us define the following Green's functions  $\langle\langle A_{l\mathbf{k}} | A_{n\mathbf{k}}^+ \rangle\rangle_E \equiv G_l^n(\mathbf{k}, E)$  and  $\langle\langle A_{l-\mathbf{k}}^+ | A_{n\mathbf{k}}^+ \rangle\rangle_E \equiv g_l^n(\mathbf{k}, E)$ . For brevity of notation we drop in the following the arguments  $\mathbf{k}$  and  $E$  at  $G_l^n(\mathbf{k}, E)$  and  $g_l^n(\mathbf{k}, E)$ . Applying the equation of motion for the above-defined Green's functions one arrives at the following set of coupled equations:

$$(E - E_1) G_1^n + SJ_{1,2} G_2^n = \frac{1}{2\pi} \delta_{l,n} \quad (4)$$

and

$$-(E + E_1) g_1^n + SJ_{1,2} g_2^n = 0 \quad (5)$$

for  $l = 1$ ,

$$(E - E_l) G_l^n + SJ_{l,l+1} G_{l+1}^n + SJ_{l,l-1} G_{l-1}^n = \frac{1}{2\pi} \delta_{l,n} \quad (6)$$

and

$$-(E + E_l) g_l^n + SJ_{l,l+1} g_{l+1}^n + SJ_{l,l-1} g_{l-1}^n = 0 \quad (7)$$

for  $1 < l < N_1$  or  $N_1 + 1 < l < N_1 + N_2 \equiv N$ ,

$$(E - E_{N_1}) G_{N_1}^n + SJ_{N_1, N_1-1} G_{N_1-1}^n + S^3 J_b (-iG_{N_1+1}^n + g_{N_1}^n + ig_{N_1+1}^n) = \frac{1}{2\pi} \delta_{N_1,n} \quad (8)$$

and

$$-(E + E_{N_1}) g_{N_1}^n + SJ_{N_1, N_1-1} g_{N_1-1}^n + S^3 J_b (ig_{N_1+1}^n + G_{N_1}^n - iG_{N_1+1}^n) = 0 \quad (9)$$

for  $l = N_1$ ,

$$(E - E_{N_1+1}) G_{N_1+1}^m + SJ_{N_1+1, N_1+2} G_{N_1+2}^m + S^3 J_b (iG_{N_1}^m - g_{N_1+1}^n + ig_{N_1}^n) = \frac{1}{2\pi} \delta_{N_1+1, n} \quad (10)$$

and

$$-(E + E_{N_1+1}) g_{N_1+1}^n + SJ_{N_1+1, N_1+2} g_{N_1+2}^n - S^3 J_b (ig_{N_1}^n + G_{N_1+1}^m + iG_{N_1}^m) = 0 \quad (11)$$

for  $l = N_1 + 1$ , and

$$(E - E_N) G_N^m + SJ_{N, N-1} G_{N-1}^m = \frac{1}{2\pi} \delta_{N, n} \quad (12)$$

and

$$-(E + E_N) g_N^n + SJ_{N, N-1} g_{N-1}^n = 0 \quad (13)$$

for  $l = N$ .

The following definitions have been introduced in the above equations:

$$E_1 = 4SJ_1[1 - \gamma_{\parallel}(\mathbf{k})] + SJ_{1,2} + g\mu_B H_1^{\text{an}}, \quad (14)$$

$$E_l = 4SJ_l[1 - \gamma_{\parallel}(\mathbf{k})] + S(J_{l, l+1} + J_{l, l-1}) + g\mu_B H_l^{\text{an}} \quad (15)$$

for  $1 < l < N_1$  or  $N_1 + 1 < l < N$ ,

$$E_{N_1} = 4SJ_{N_1}[1 - \gamma_{\parallel}(\mathbf{k})] + SJ_{N_1, N_1-1} - S^3 J_b + g\mu_B H_{N_1}^{\text{an}}, \quad (16)$$

$$E_{N_1+1} = 4SJ_{N_1+1}[1 - \gamma_{\parallel}(\mathbf{k})] + SJ_{N_1+1, N_1+2} - S^3 J_b + g\mu_B H_{N_1+1}^{\text{an}}, \quad (17)$$

and

$$E_N = 4SJ_N[1 - \gamma_{\parallel}(\mathbf{k})] + SJ_{N, N-1} + g\mu_B H_N^{\text{an}}. \quad (18)$$

For each value of  $n$  ( $n = 1, \dots, N$ ) equations (4) to (13) form a set of  $2N$  equations for  $2N$  unknown Green's functions  $G_l^m$  and  $g_l^n$ . For  $J_b = 0$  this system of equations reduces to two independent systems, for  $G_l^m$  and  $g_l^n$ . In the case of  $J_b \neq 0$ , however, all equations are coupled.

Spin-wave excitations correspond to poles of the Green's functions. Those poles can be found from the secular equation corresponding to the homogeneous part of the set of equations (4) to (13).

In the following the spin-wave spectrum will be analysed quantitatively in the case when both films are magnetically identical and spatially uniform (except the surface and interface atomic planes). Accordingly, we assume that all exchange integrals are the same except those between spins occupying the surface or interface atomic planes, i.e.,  $J_1 = J_N \equiv J^{\text{surf}}$ ,  $J_{N_1} = J_{N_1+1} \equiv J^{\text{int}}$ ,  $J_l = J$  for  $2 \leq l \leq N_1 - 1$  and for  $N_1 + 2 \leq l \leq N - 1$ ,  $J_{l, l+1} = J$  for  $1 \leq l \leq N_1 - 1$  or  $N_1 + 1 \leq l \leq N - 1$ . Similarly, we assume  $H_l^{\text{an}} = H_{\text{surf}}^{\text{an}}$  for the two surface atomic planes ( $l = 1, N$ ),  $H_l^{\text{an}} = H_{\text{int}}^{\text{an}}$  for the interface atomic planes ( $l = N_1, N_1 + 1$ ) and  $H_l^{\text{an}} = H^{\text{an}}$  for the other atomic planes. Two cases will be analysed in detail, (i) the case when the numbers  $N_1$  and  $N_2$  are very large, so one can consider both films as semi-infinite systems ( $N_1, N_2 \rightarrow \infty$ ), and (ii) the case when the numbers  $N_1$  and  $N_2$  are rather small (films consisting of a few or few tens of atomic planes).

#### 4. Limit of Semi-Infinite Ferromagnets

Consider now the situation when the films are thick enough to be treated as bulk systems. The corresponding spin-wave spectrum consists then of two parts, spin waves of bulk character and modes which are exponentially localized at the interface. Consider first briefly the bulk modes. Those are the modes which propagate across the ferromagnets as plane waves with real components of the corresponding wavevector. At the interface they are usually partially reflected and transmitted. The corresponding dispersion relation is the same as for spin waves in a single bulk ferromagnet,

$$\begin{aligned} E(\mathbf{k}, k_{\perp}) &= 4SJ[1 - \gamma_{\parallel}(\mathbf{k})] + 2SJ + g\mu_{\text{B}}H^{\text{an}} - 2SJ \cos k_{\perp}a \\ &= 6SJ[1 - \gamma(\mathbf{q})] + g\mu_{\text{B}}H^{\text{an}}, \end{aligned} \quad (19)$$

where  $k_{\perp}$  is the wavevector component perpendicular to the interface,  $\mathbf{q}$  is the three-dimensional wavevector,  $\mathbf{q} = (\mathbf{k}, k_{\perp})$ , and  $\gamma(\mathbf{q}) = (1/3)(\cos q_x a + \cos q_y a + \cos q_z a)$ . The upper band edge is determined by

$$E(\mathbf{k}, k_{\perp} = \pi/a) = 4SJ[1 - \gamma_{\parallel}(\mathbf{k})] + 4SJ + g\mu_{\text{B}}H^{\text{an}}, \quad (20)$$

while the lower band edge is described by the expression

$$E(\mathbf{k}, k_{\perp} = 0) = 4SJ[1 - \gamma_{\parallel}(\mathbf{k})] + g\mu_{\text{B}}H^{\text{an}}. \quad (21)$$

The band of bulk modes is represented in Fig. 1 by the shaded area. The spin-wave spectrum is shown there as a function of the parameter  $\Lambda$ , which is defined as

$$\Lambda = 1 - \gamma_{\parallel}(\mathbf{k}). \quad (22)$$

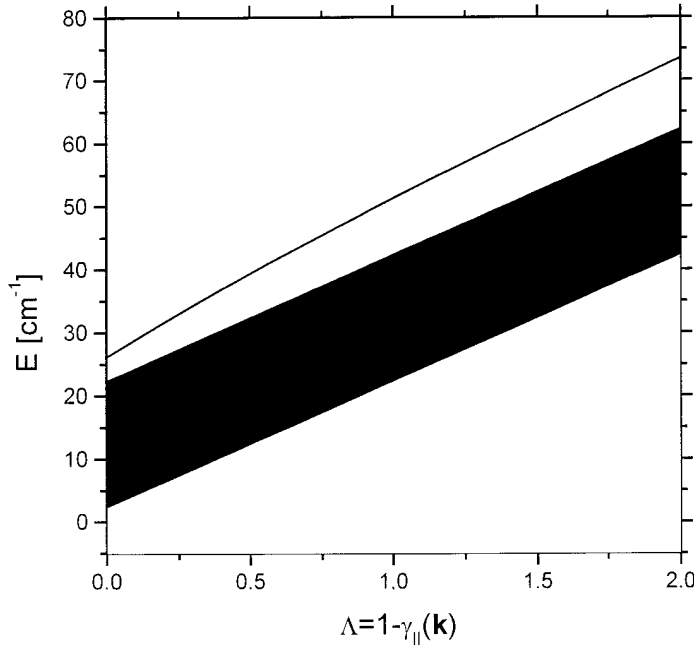


Fig. 1. Spectrum of bulk (shaded region) and interfacial (solid line) modes in the case of two identical semi-infinite ferromagnets, plotted as a function of the parameter  $\Lambda = 1 - \gamma_{\parallel}(\mathbf{k})$ . The parameters assumed here are  $J = J^{\text{int}} = 2.5 \text{ cm}^{-1}$ ,  $J_{\text{b}} = -1 \text{ cm}^{-1}$ ,  $H_{\text{int}}^{\text{an}} = H^{\text{an}} = 2.5 \text{ T}$ ,  $S = 2.5$  and  $g = 2$

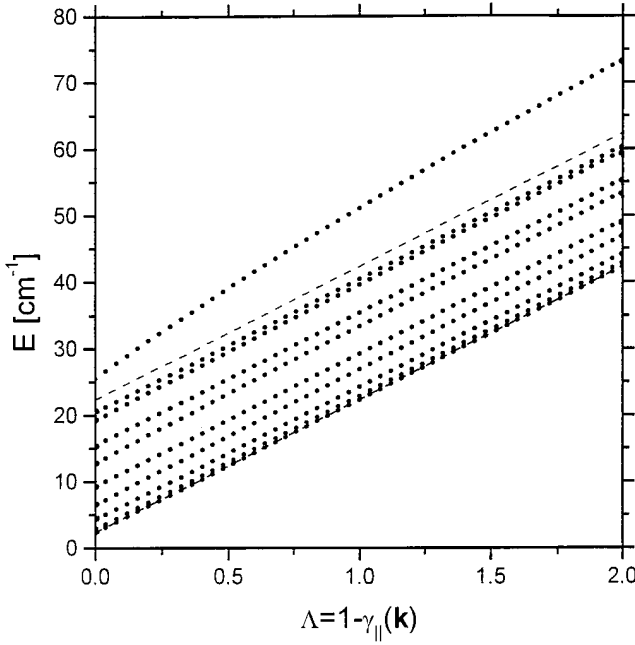


Fig. 2. Spin-wave spectrum as a function of the parameter  $\Lambda = 1 - \gamma_{\parallel}(\mathbf{k})$  for two identical films consisting of  $N_1 = N_2 = 5$  atomic planes. The other parameters are  $J = J^{\text{int}} = J^{\text{surf}} = 2.5 \text{ cm}^{-1}$ ,  $J_b = -1 \text{ cm}^{-1}$ ,  $H_{\text{int}}^{\text{an}} = H_{\text{surf}}^{\text{an}} = H^{\text{an}} = 2.5 \text{ T}$ ,  $S = 2.5$  and  $g = 2$ . The dashed curves correspond to the upper and lower band edges of the corresponding spectrum of bulk modes (see Fig. 1)

The system can also support modes which are localized exponentially at the interface. One can show that those modes correspond to solutions of the following equation:

$$\begin{aligned} [E^2 - (E^{\text{int}} - SJx)^2]^2 + 2(S^3 J_b)^2 [E^2 - (E^{\text{int}} - SJx)^2] \\ - 4(S^3 J_b)^2 (E^{\text{int}} - SJx + S^3 J_b)^2 + (S^3 J_b)^4 = 0, \end{aligned} \quad (23)$$

where  $x (|x| < 1)$  is the decay coefficient,

$$x = \frac{1}{2SJ} (E_0 - E \pm [(E_0 - E)^2 - (2SJ)^2]^{1/2}). \quad (24)$$

In the above equations  $E^{\text{int}}$  and  $E_0$  are defined as

$$E^{\text{int}} = 4SJ^{\text{int}}[1 - \gamma_{\parallel}(\mathbf{k})] + SJ - S^3 J_b + g\mu_B H_{\text{int}}^{\text{an}}, \quad (25)$$

$$E_0 = 4SJ[1 - \gamma_{\parallel}(\mathbf{k})] + 2SJ + g\mu_B H^{\text{an}}. \quad (26)$$

Equations (23) and (24) have been solved numerically. Only those solutions which correspond to  $|x| < 1$  can describe real interfacial modes. When  $x > 0$ , the interfacial modes are of acoustic type, while for  $x < 0$  they are of optical type. The appropriate numerical results are shown in Fig. 1 by the solid line. For the parameters assumed in Fig. 1 there is only one interfacial mode which is of optical type.

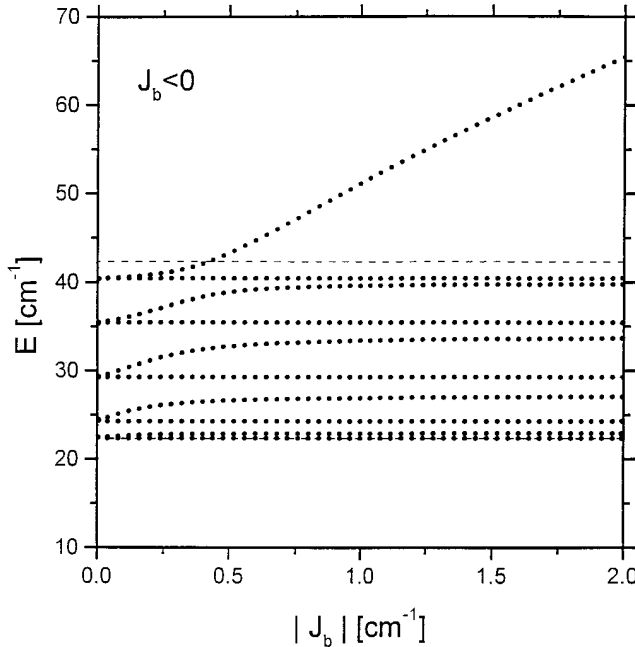


Fig. 3. Spin-wave spectrum for a bilayer, plotted as a function of the absolute value  $|J_b|$  of the interlayer coupling parameter  $J_b$  and calculated for  $\mathcal{A} = 1$ . The other parameters are the same as in Fig. 2. The dashed curves correspond to the upper and lower band edges of the corresponding bulk spectrum

## 5. Thin Exchange-Coupled Bilayers

Consider now the case of two thin films which are coupled at the interface by the biquadratic exchange. The corresponding spin-wave spectrum was obtained by numerical solution of the corresponding secular equation. In Fig. 2 we show the spectrum in the case of two identical films, each consisting of five atomic planes,  $N_1 = N_2 = 5$ . As in Fig. 1, the spin-wave spectrum is shown there as a function of the parameter  $\mathcal{A}$  defined by Eq. (22). The dashed curves in Fig. 2 represent the lower and upper band edges of the corresponding spectrum of bulk modes (see the shaded region in Fig. 1). For the parameters assumed in Fig. 2 one of the ten modes turns into an interfacial mode, which is above the spectrum of bulk modes. In Fig. 3 we see the same spin-wave spectrum, but shown as a function of the absolute value of the parameter  $J_b$  (we recall that  $J_b$  is negative,  $J_b < 0$ ). For  $J_b = 0$  both thin films are decoupled and therefore each mode is doubly degenerate. This degeneration is lifted by the interlayer coupling ( $|J_b| > 0$ ).

## 6. Summary

We have analysed bulk and interfacial spin waves in a system composed of two semi-infinite ferromagnets, as well as in a structure consisting of two thin ferromagnetic films. The semi-infinite ferromagnets (and also the thin films) were coupled at the interface by a biquadratic exchange interaction. In the ground state configuration the magnetizations of both ferromagnetic systems were assumed to be parallel to the interface, but perpen-

dicular to one another. In the case of two semi-infinite systems the spin-wave spectrum consists of interfacial modes and a band of bulk modes. For thin bilayers, the number of different modes is equal to the number of atomic planes. Those modes usually have bulk character. However, for some values of the interfacial parameters, interface modes can also occur.

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