Eddy currents and spin excitations in conducting ferromagnetic films

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We explore the influence of the finite conductivity on spin waves in metallic ferromagnetic films. We consider propagation perpendicular to the magnetization, which is parallel to the surface, and wavelengths sufficiently long that the influence of exchange may be ignored. Precession of the magnetization induces eddy currents which damp the spin waves, and also renormalize the dispersion relation of the Damon-Eshbach mode encountered in this geometry. We provide analytic formulas which describe these effects, in various limits. Studies through use of a Green's-function method explore the influence of the conductivity on the spectrum of spin fluctuations, in various wavelength regimes.

I. INTRODUCTION

The theory of spin-wave excitations in ferromagnetic films is a classic topic in magnetism, treated theoretically in various limits many years ago.[1] Experimentally, these modes may be probed by ferromagnetic resonance, or by the Brillouin scattering of light.[2] In such experiments, the modes excited have wavelengths very long compared to a lattice constant. In this regime, exchange interactions contribute negligibly to the excitation energy. Such spin waves are then described accurately by a theory based on magnetostatics, not only for films, but for samples of diverse shapes.[3]

Interest in this area has revived in recent years, as a consequence of modern sample preparation techniques, which allow the preparation of very thin ferromagnetic films and multilayers of extraordinary quality, on diverse substrates.[2,4]

The early studies of spin waves in thin films, spheres or ellipsoids, were directed largely toward insulating ferrites, such as YIG.[3] The new materials described in the previous paragraph incorporate films of ferromagnetic metals, such as Fe. Theoretical descriptions of spin excitations directed toward these materials include features such as the anisotropies, dipolar couplings and interfilm exchange couplings found in these samples,[5–8] but do not explicitly acknowledge the fact that the constituent films are metallic in nature. It is the purpose of this paper to present and explore the influence of finite conductivity on the spin-wave excitations of a ferromagnetic film.

The issue of concern is the following. When a spin wave is excited, of course the magnetization precesses at each point in space, generally on a trajectory of elliptical character. The precessing magnetization generates a time-dependent internal magnetic induction $\mathbf{b}(\mathbf{x},t)$ everywhere. By Faraday's Law, this time-dependent magnetic induction must induce an electric field $\mathbf{e}(\mathbf{x},t)$. If the conductivity is finite, eddy currents are generated by this electric field. The ohmic dissipation associated with the eddy currents is a source of linewidth for the spin-wave mode. We show below that in addition, the eddy currents can renormalize the dispersion relation of the modes.

It is of interest to inquire if the films and multilayers can be utilized for device applications. The lifetime of the spin-wave modes is a critical parameter when such applications are considered. We show below that in certain regimes of wavelength and film thickness, eddy current damping becomes very severe indeed. It is also the case, however, that for film thicknesses and wavelengths examined in numerous recent experiments, its influence is modest.

We confine our attention to a geometry encountered commonly. We consider a ferromagnetic film with magnetization parallel to the surfaces, and we consider spin waves which propagate perpendicular to the magnetization. This is the case studied in most current experiments.[2] In this geometry, one encounters the Damon-Eshbach wave, a mode which in the limit of wavelength short compared to the sample thickness becomes a surface spin wave,[1] bound to either the upper or lower film surface, depending on its direction of propagation. The methods used here are readily extended to other magnetization orientations, or propagation directions.

We begin our discussion in Sec. II with a derivation of the dispersion relation of the Damon-Eshbach wave in the presence of finite conductivity. One may extract from this information on the linewidth of the mode by extracting the imaginary part of the frequency, for a given wave vector $k_\parallel$ parallel to the surface. We are led to simple, useful analytic formulas here, in special limits.

It is difficult to extract useful information from the implicit dispersion relation, in regimes where eddy current damping and renormalization effects are severe. Thus, in Sec. III, we derive a set of Green's functions which may be used to describe the response of the metallic ferromagnet to an arbitrary external microwave field, applied in the plane perpendicular to the magnetization. These response functions can be used for diverse purposes. By invoking the fluctuation-dissipation theorem, for example, we can use them to explore the frequency spectrum of thermal spin fluctuations, and also to describe the Brillouin spectrum of the film.[5,6] By such a study, we extract information on the nature of the spin excitations, in the frequency and wavelength regime where the influence of eddy currents is severe.

In the analysis presented here, we ignore the influence of exchange. The precessing moments, in the present picture, generate dipolar fields which influence the dispersion relation of the spin waves we consider. Under the conditions explored here, exchange effects on the Damon-Eshbach waves are quite modest, and may be set aside with little

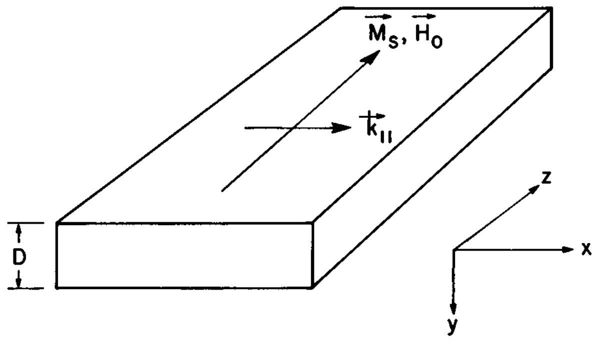

FIG. 1. The geometry considered in the present paper. We have a ferromagnetic film of thickness D, with magnetization $\mathbf{M}_s$ parallel to the surface. An external magnetic field $\mathbf{H}_0$ is applied parallel to the surface. We consider spin waves, with wave vector $\mathbf{k}_\parallel$ that propagate parallel to the x axis.

consequence. One may appreciate this from earlier discussions, in which exchange is included fully.[5] In the particular case of Brillouin spectra, exchange influences the spectrum of standing spin waves importantly, by introducing splittings between the various standing wave modes. This feature is absent from the calculations presented below. It is straightforward, in principle, to extend earlier discussions[5] to include both exchange and also the eddy current effects explored here. The cost in complexity is substantial. We leave this for future work, when we wish quantitative contact between theory and experiment, in the standing spin-wave spectra of conducting films.

II. THE INFLUENCE OF FINITE CONDUCTIVITY ON THE DISPERSION RELATION OF DAMON-ESHBACH WAVES

The geometry we consider is illustrated in Fig. 1. We have a ferromagnetic film of thickness D, with magnetization $\mathbf{M}_s$ parallel to the surface. An external magnetic field $\mathbf{H}_0$ is applied parallel to $\mathbf{M}_s$. The coordinate system is aligned so $\mathbf{M}_s$ is along the z axis, the film lies between $y=0$ and $y=D$, and the waves we consider propagate in the x direction. Their wavelength will be sufficiently long that we ignore exchange effects.

The spin waves have frequency Ω. As remarked in Sec. I, the precessing magnetization generates magnetic field $\mathbf{h}$ and a magnetic induction $\mathbf{b}$, both of which oscillate in time with the frequency Ω. Both $\mathbf{h}$ and $\mathbf{b}$ lie in the xy plane, for the geometry in Fig. 1. For the ferromagnet, we have the constitutive relations[9]

$$b_x=\mu_1 h_x+i\mu_2 h_y \tag{2.1a}$$

and

$$b_y=\mu_1 h_y-i\mu_2 h_x, \tag{2.1b}$$

where

$$\mu_1=1+\frac{4\pi\Omega_M\Omega_H}{\Omega_H^2-(\Omega+i/\tau)^2} \tag{2.2a}$$

and

$$\mu_2=\frac{4\pi\Omega_M(\Omega+i/\tau)}{\Omega_H^2-(\Omega+i/\tau)^2}. \tag{2.2b}$$

Here $\Omega_H=\gamma H_0$ and $\Omega_M=\gamma M_s$, while τ is a phenomenological relaxation time. Within the film, we have

$$\nabla\times\mathbf{h}=\frac{4\pi\sigma}{c}\,\mathbf{e}, \tag{2.3a}$$

$$\nabla\cdot\mathbf{b}=0, \tag{2.3b}$$

and also

$$\nabla\times\mathbf{e}=i\,\frac{\Omega}{c}\,\mathbf{b}, \tag{2.3c}$$

where $\mathbf{e}$ is the electric field generated by the precessing magnetization. The conductivity of the medium is σ. For the geometry in Fig. 1, the electric field $\mathbf{e}$ is parallel to the z direction. We have ignored the displacement current term on the right-hand side of Eq. (2.3a), an approximation valid so long as we are concerned with lengths smaller c/Ω. The retardation effects introduced by this term will be negligible for the examples explored here.

All fields in the above equations are proportional to $\exp(ik_\parallel x)$, with a y dependence to be determined. It is a short exercise to find a pair of equations obeyed by h_x and h_y:

$$\left[ik_\parallel^2+\frac{2\mu_1}{\delta_0^2}\right]h_y-\left[k_\parallel\,\frac{\partial}{\partial y}+i\,\frac{2\mu_2}{\delta_0^2}\right]h_x=0 \tag{2.4a}$$

and

$$i\left[\mu_1 k_\parallel-\mu_2\,\frac{\partial}{\partial y}\right]h_x+\left[\mu_1\,\frac{\partial}{\partial y}-\mu_2 k_\parallel\right]h_y=0. \tag{2.4b}$$

In these equations, we have introduced the classical skin depth δ_0, in a medium with conductivity σ:

$$\delta_0=\frac{c}{(2\pi\sigma\Omega)^{1/2}}. \tag{2.5}$$

We seek solutions in the medium with the spatial variation $\exp[\pm Qy]$. One finds, after a brief calculation,

$$Q=\left\{k_\parallel^2+\frac{2\mu_V}{i\delta_0^2}\right\}^{1/2}, \tag{2.6}$$

where here and elsewhere in the paper we choose $\mathrm{Re}(Q)>0$. The quantity

$$\mu_V=\frac{\mu_1^2-\mu_2^2}{\mu_1}=\frac{\Omega_B^2-(\Omega+i/\tau)^2}{\Omega_H\Omega_B-(\Omega+i/\tau)^2}. \tag{2.7}$$

In Eq. (2.7), $\Omega_B=\Omega_H+4\pi\Omega_M=\gamma(H_0+4\pi M_S)$.

Some comments on the physical content of Eq. (2.6) are in order. First of all, in a conducting material, the expression in Eq. (2.5) is the well-known classical skin depth. In a material with a nonzero magnetic permeability, the skin depth is modified by the magnetic response. The combination

$$\delta_{\rm FM}=\frac{\delta_0}{\sqrt{\mu_V}} \tag{2.8}$$

which appears in Eq. (2.6) is the effective skin depth in the ferromagnet. Clearly, this is strongly frequency dependent, in the vicinity of the resonances in the structure. Recall that $(\Omega_H\Omega_B)^{1/2}$ is the ferromagnetic resonance frequency of a thin film with magnetization parallel to the surface.[10] Near resonance, μ_V becomes very large, and the skin depth is reduced dramatically from the classical value δ_0. If ΔH is the linewidth, defined as the full width at half maximum of the absorption line (then in our phenomenology $\Delta H=2/\tau$), on resonance the skin depth is well approximated by

$$\delta_{\rm FM}(\Omega=(\Omega_H\Omega_B)^{1/2})\equiv\delta_R=\delta_0\left(\frac{\gamma\Delta H}{4\pi\Omega_M}\right)^{1/2}\left(\frac{\Omega_H}{\Omega_B}\right)^{1/4}. \tag{2.9}$$

In Fe, $4\pi M_s\cong 2.1\times10^4$ G. If the linewidth $\Delta H=100$ G, and the resonance frequency is in the 10 GHz range, δ_R is smaller than δ_0 by a factor of roughly 25. The influence of the spin system on the penetration depth of electromagnetic radiation is thus dramatic, near resonance. The skin depth $\delta_{\rm FM}$ becomes very large near $\Omega=\Omega_B$. The film ''opens up,'' and its transmissivity increases dramatically. This is the phenomenon of ''antiresonance,'' discovered some years ago.[11]

We recover the theory appropriate to insulating media in the limit $\delta_0\to\infty$. Then $Q=|k_\parallel|$. Suppose we consider an infinitely thick film, $D\to\infty$, and a Damon-Eshbach wave propagating down its surface. The spatial variation of the disturbances associated with the wave is controlled by $\exp(-Qy)$, which becomes $\exp(-|k_\parallel|y)$, as $\delta_0\to\infty$. As the wave vector $k_\parallel\to0$, the fields penetrate ever more deeply into the material, in a manner identical to Rayleigh surface acoustic waves.[12] This behavior is modified dramatically by the presence of the finite conductivity, where now

$$\lim_{k_\parallel\to0}Q=\left(\frac{2}{i\,\delta_{\rm FM}}\right)^{1/2}. \tag{2.10}$$

The fields can penetrate no deeper than the skin depth $\delta_{\rm FM}$ associated with the frequency of the wave. When $D\gg\delta_{\rm FM}$, in the long-wavelength limit, the fields of the Damon-Eshbach wave will be confined to a channel of depth $\delta_{\rm FM}$, near the surface. Clearly, in this regime, the eddy currents not only limit the lifetime of the wave, but will modify its effective dispersion relation dramatically as well.

It is a straightforward matter, following procedures now standard, to determine the implicit dispersion relation of the wave. Inside the film, the various fields are supposed a superposition of $\exp(+Qy)$, and $\exp(-Qy)$. These are matched to fields in the vacuum through standard boundary conditions (conservation of h_x and b_y). For $y>D$, all fields vary as $\exp(-k_\parallel(y-D))$ and for $y<0$, they scale as $\exp(+k_\parallel y)$.

It should be remarked that here, and throughout the remainder of the paper, we suppose $k_\parallel>0$. The frequency Ω can then be either positive or negative.[13] Disturbances which propagate from left to right are described by choosing $\Omega>0$, and those which propagate from right to left by choosing $\Omega<0$.

When the analysis is completed, we find the implicit dispersion relation may be written

$$\frac{(Q+\mu_V k_\parallel)^2-(\mu_2/\mu_1)^2k_\parallel^2}{(Q-\mu_V k_\parallel)^2-(\mu_2/\mu_1)^2k_\parallel^2}=\exp(-2QD). \tag{2.11}$$

This may be rearranged to read

$$(\Omega+i/\tau)^2=\Omega_H\Omega_B+\frac{8\pi^2\Omega_M^2k_\parallel^2\tanh(QD)}{k_\parallel Q+(k_\parallel^2-i/\delta_0^2)\tanh(QD)}, \tag{2.12}$$

which as $\delta_0\to\infty$, yields the standard dispersion relation[1] appropriate to the insulating film (for $\tau\to\infty$):

$$\Omega^2(k_\parallel)=\gamma^2(H_0+2\pi M_s)^2-4\pi^2\gamma^2M_s^2\exp(-2k_\parallel D). \tag{2.13}$$

From Eq. (2.12), which remains an implicit dispersion relation by virtue of the presence of the frequency-dependent quantity Q on the right-hand side, we can extract simple formulas in special limits. To treat the regime $k_\parallel\delta_0\gg1$, for example, we replace Q by $Q\cong k_\parallel[1-i\mu_V/(k_\parallel\delta_0)^2+\cdots]$ and expand the right-hand side in powers of $(k_\parallel\delta_0)^{-2}$, retaining only the leading term. After some algebra, we obtain (we let $\tau\to\infty$ for the moment)

$$\begin{aligned}\Omega^2=&\gamma^2(H_0+2\pi M_s)^2-4\pi^2\gamma^2M_s^2\exp(-2QD)\\&-\frac{2\pi i\gamma^2M_s}{(k_\parallel\delta_0)^2}[(H_0+2\pi M_s)+(H_0+6\pi M_s)e^{-2QD}]\\&+\cdots.\end{aligned} \tag{2.14}$$

The result in Eq. (2.13) remains an implicit dispersion relation, by virtue of the presence of Q on the right-hand side. There are two limits where we may generate simple results: the very thin film limit $D\to0$, and the thick film limit $D\to\infty$. We have, restoring the influence of the relaxation time τ,

$$\lim_{D\to\infty}\Omega(k_\parallel)=\gamma(H_0+2\pi M_s)-\frac{i}{\tau}-\frac{i\pi\gamma M_s}{(k_\parallel\delta_0)^2}+\cdots, \tag{2.15a}$$

$$\lim_{D\to0}\Omega(k_\parallel)=\gamma(H_0B)^{1/2}-\frac{i}{\tau}-\frac{2i\pi\gamma M_s}{(k_\parallel\delta_0)^2}\left(\frac{B}{H}\right)+\cdots. \tag{2.15b}$$

Clearly, the last terms in Eqs. (2.14) describe damping produced by the eddy currents induced by the spin motion.

We may also extract the behavior of the effective dispersion relation for small $k_\parallel$, from Eq. (2.11). Quite clearly, as $k_\parallel \rightarrow 0$, Ω tends to $(\Omega_H \Omega_B)^{1/2} - i/\tau$, the (damped) ferromagnetic resonance frequency of the film. The task of extracting the first correction to this term in tricky, in the small $k_\parallel$ limit. For $k_\parallel \delta_0 \ll 1$, clearly we may ignore $k_\parallel^2$ compared to i/δ_0^2. Also, in Eq. (2.11), we replace Q by simply $Q \cong (2\mu_V / i\delta_0^2)^{1/2}$. Note that as $k_\parallel \rightarrow 0$, and $(\Omega + i/\tau) \rightarrow (\Omega_H \Omega_B)^{1/2}$, μ_V and thus Q become very large. We shall assume $QD \rightarrow \infty$ as $k_\parallel \rightarrow 0$, so $\tanh(QD) \rightarrow 1$. The criterion for the validity of this assumption will be stated when the analysis is complete. We then have

$$\left(\Omega + \frac{i}{\tau}\right)^2 \cong \Omega_H \Omega_B + \frac{8\pi^2 \Omega_M^2 (k_\parallel \delta_0)^2}{[k_\parallel \delta_0 (2\mu_V / i)^{1/2} - i]}. \qquad (2.16)$$

We seek a solution of Eq. (2.15) with

$$(\Omega + i/\tau)^2 = \Omega_H \Omega_B - iA(k_\parallel \delta_0)^2, \qquad (2.17)$$

where A is to be determined. Now

$$\mu_V = \frac{\Omega_B^2 - (\Omega + i/\tau)^2}{\Omega_B \Omega_H - (\Omega + i/\tau)^2} \cong \frac{4\pi \Omega_M \Omega_B}{iA(k_\parallel \delta_0)^2}, \qquad (2.18)$$

as $k_\parallel \rightarrow 0$. When Eq. (2.16) and Eq. (2.17) are inserted into Eq. (2.15), we find two solutions for A:

$$A_\pm = 2\pi \Omega_M (\Omega_B^{1/2} \pm \Omega_H^{1/2}). \qquad (2.19)$$

We then have, for each choice of wave vector, two propagating modes, each characterized by a different lifetime and propagation length:

$$\Omega_\pm(k_\parallel) = \gamma (H_0 B)^{1/2} - \frac{i}{\tau} - i\pi\gamma M_S \left\{ \left(\frac{B}{H_0}\right)^{1/2} \pm \left(\frac{H_0}{B}\right)^{1/2} \right\} \times (k_\parallel \delta_0)^2 + \cdots . \qquad (2.20)$$

The origin of the two-mode behavior is the resonance in the Voigt susceptibility μ_V which controls the effective skin depth. A discussion of the excitation of these modes requires an analysis of any particular experiment of interest, to assess their relative amplitude. The Green's functions discussed in Sec. III will allow one to perform such analyses, for any desired excitation scheme.

We remark on the requirement for our assumption $|QD| \gg 1$ to be valid. From Eq. (2.17), as $k_\parallel \rightarrow 0$, one sees

$$|QD| = \frac{D}{k_\parallel \delta_0^2} \left(\frac{8\pi \Omega_M \Omega_B}{A}\right)^{1/2}, \qquad (2.21)$$

so we must have $D \gg k_\parallel \delta_0^2$, or $k_\parallel \delta_0 \ll (D/\delta_0)$.

The results in Eqs. (2.14) and Eq. (2.19) allow one to estimate the influence of eddy current damping, for various experimental situations of interest. For instance, in typical Brillouin-scattering experiments, the modes excited have $k_\parallel \cong 10^5$ cm^{-1}, while for metallic Fe, in the microwave frequency range (10 GHz, for example), one finds $\delta_0 \cong 10^{-4}$ cm. We thus have $k_\parallel \delta_0 \cong 10$ in such studies, and we may use Eqs. (2.14) to estimate the eddy current damping effects. If one recalls $4\pi M_s = 21$ kG for Fe, one sees the eddy current contribution to the linewidth is in the range of a few tens of Gauss. This is below current experimental resolution, but is by no means negligible.

The limit $k_\parallel \delta_0 \ll 1$ applies to ferromagnetic resonance experiments. One sees from Eqs. (2.19) that $k_\parallel$ must be finite for eddy current damping to influence the linewidth. This suggests that in most such experiments, eddy current damping should be very small. We expect $k_\parallel \cong \pi/W$, with W the width of the sample, if one has spin pinning at the edges.

The approximate formulas obtained above suggest that when $k_\parallel \delta_0 \cong 1$, the eddy current damping should be very substantial. It is difficult to extract meaningful information from the effective dispersion relation, in this regime. Thus, in Sec. III we turn to a discussion of the Green's-function method used in numerous earlier papers. This method can provide information on the excitations in the system, even in the limit of strong damping.

III. GREEN'S-FUNCTION DESCRIPTION OF THE FILM RESPONSE IN THE PRESENCE OF EDDY CURRENTS

In Sec. II, we examined the influence of eddy currents on the dispersion relation of the magnetostatic spin waves of the ferromagnetic film, for the geometry illustrated in Fig. 1. We saw that the eddy currents introduce damping, which could be very severe in the wave-length regime where $k_\parallel \delta_0 \sim 1$, if the asymptotic formulae of Sec. II are extrapolated into this regime. In principal, at least, we could explore this issue further through examination of the dispersion relation found in Sec. II, through numerical studies which trace out the complex frequency as a function of wave vector. In the regime where damping is strong, such studies are of limited usefulness, since it is often unclear how the dispersion relation or damping rate rate which emerges are related to various experimental probes of the system. We recall earlier discussions which led to unphysical conclusions in other contexts,[14] and as a consequence we turn to the development of a Green's-function technique within which strong dissipative effects can be incorporated, and related to diverse experiments in an unambiguous manner.

One proceeds by supposing the film is driven by a weak external magnetic field in the xy plane, given by

$$\mathbf{h}^{(e)}(\mathbf{x},t) = [\hat{x} h_x^{(e)}(y) + \hat{y} h_y^{(e)}(y)] \exp(ik_\parallel x - i\Omega t), \qquad (3.1)$$

where the profile of the external field in the y direction is arbitrary. In response to this field, in linear-response theory, the magnetization of the film is

$$\mathbf{M}(\mathbf{x},t) = \hat{z} M_s + \mathbf{m}(\mathbf{x},t), \qquad (3.2a)$$

where

$$\mathbf{m}(\mathbf{x},t) = [\hat{x} m_x(y) + \hat{y} m_y(y)] \exp(ik_\parallel x - i\Omega t). \qquad (3.2b)$$

The elements of the external field $(h_x^{(e)}(y), h_y^{(e)}(y))$ are related to the system response $(m_x(y), m_y(y))$ by a matrix of Green's functions introduced below.

We consider the system driven by an external perturbation characterized by only a single frequency Ω, and wave vector $k_\parallel$ parallel to the surface. Within linear-response theory, straightforward Fourier synthesis may be used to describe the response of the film to a perturbation of arbitrary form in space and time. By this means, for example, with the use of our Green's functions, one can develop the theory of spin-wave generation by a current bearing strip or meander line (modeled as a periodic structure) deposited on the surface. In this paper, we do confine our attention to propagation perpendicular to the magnetization, as illustrated in Fig. 1.

The fluctuation-dissipation theorem also may be employed to relate our Green's functions to the amplitude of thermal spin fluctuations in the film, of frequency Ω and wave vector $k_\parallel$. The spatial variation of the thermal spin fluctuations may be explored by this means. Brillouin-scattering experiments provide us with a probe of such thermal spin fluctuations, within the optical skin depth. Thus, our earlier theories of the Brillouin spectrum of ferromagnetic films and superlattices utilized similar Green's functions for this purpose.[5,6] One may view the present paper as an extension of an earlier description[5] of spin waves in thin films to include the influence of eddy currents induced by the fluctuating magnetization. The earlier study incorporated exchange effects ignored here; as noted in Sec. I the present discussion is readily extended to include exchange, at a cost in technical complexity. Our interest will center on the Damon-Eshbach portion of the response presently; exchange effects on this mode are quite modest, in the experiments which motivate our analysis.

We now turn to the formalism. As noted above, our task is to generate a description of the response of the film when it is driven by the external magnetic field in Eq. (3.1). The basic equation we solve describes the precession of the magnetization of the film in an external field:

$$\frac{d\mathbf{m}}{dt}=-i\Omega\mathbf{m}=\gamma(\mathbf{M}\times\mathbf{h}), \tag{3.3}$$

where in the spirit of spin-wave theory, we linearize the right-hand side of Eq. (3.3) with respect to the fluctuating portion of the magnetization defined in Eqs. (3.2). The quantity γ is the gyromagnetic ratio. The magnetic field $\mathbf{h}$ in Eq. (3.3) is the externally applied field $\mathbf{h}^{(e)}(\mathbf{x},t)$ described in Eq. (3.1), and to this is added the fluctuating dipolar field $\mathbf{h}^{(d)}(\mathbf{x},t)$ generated by $\mathbf{m}(\mathbf{x},t)$. The dipolar field is linear in $\mathbf{m}$, and is calculated by solving Maxwell's equation. We pose the problem of generating formulas for $m_x(y), m_y(y)$ in Eq. (3.2b) when $h_x^{(e)}(y), h_y^{(e)}(y)$ in Eq. (3.1) are arbitrary, unspecified functions of y. All quantities exhibit the time dependence $\exp(-i\Omega t)$, and vary with x as $\exp(ik_\parallel x)$. The dipolar field $\mathbf{h}^{(d)}$ also lies in the xy plane for the geometry considered, and the eddy currents are parallel to the z direction. Thus, the electric field is parallel to z also.

We define $\Omega_H=\gamma H_0$, and $\Omega_M=\gamma M_s$. The magnetization components then are found from

$$i\Omega m_x(y)+\Omega_H m_y(y)-\Omega_M h_y^{(d)}(y)=\Omega_M h_y^{(e)}(y) \tag{3.4a}$$

and

$$-\Omega_H m_x(y)+i\Omega m_y(y)+\Omega_M h_x^{(d)}(y)=-\Omega_M h_x^{(e)}(y), \tag{3.4b}$$

while after elimination of the electric field, we may generate the dipolar field from

$$4\pi i\lambda^2 m_x(y)+\left[\frac{\partial^2}{\partial y^2}+i\lambda^2\right]h_x^{(d)}(y)-ik_\parallel\frac{\partial h_y^{(d)}(y)}{\partial y}=0 \tag{3.4c}$$

and

$$4\pi i\lambda^2 m_y(y)-ik_\parallel\frac{\partial h_x^{(d)}(y)}{\partial y}-[k_\parallel^2-i\lambda^2]h_y^{(d)}(y)=0. \tag{3.4d}$$

In these last two equations, $\lambda^2=2/\delta_0^2$, where δ_0 is the classical skin depth defined in Eq. (2.5). The processing magnetization generates fields in the vacuum, which must be matched to those in the film through appropriate boundary conditions. These are conservation of tangential $\mathbf{h}$ [continuity of $h_x^{(d)}(y)$ at the film surfaces], and normal $\mathbf{b}$ [continuity of $h_y^{(d)}(y)+4\pi m_y(y)$]. The fields in the vacuum are described by Eqs. (3.4c) and (3.4d) with λ^2 set to zero.

We rewrite Eqs. (3.4) by introducing two four component vectors.

$$u=\begin{pmatrix} m_x(y) \\ m_y(y) \\ h_x^{(d)}(y) \\ h_y^{(d)}(y)\end{pmatrix} \tag{3.5}$$

and

$$f=\begin{pmatrix} \Omega_M h_y^{(e)}(y) \\ -\Omega_M h_x^{(e)}(y) \\ 0 \\ 0\end{pmatrix}, \tag{3.6}$$

so they acquire the form

$$\sum_{j=1}^{4} L_{ij}u_j(y)=f_i(y) \tag{3.7}$$

with L_{ij} a 4×4 matrix of differential operators. We obtain the solution of Eqs. (3.7) by introducing a matrix of Green's functions $G_{ij}(y,y')$ which satisfy

$$\sum_{j=1}^{4} L_{ik}G_{kj}(y,y')=\delta_{ij}\delta(y-y'). \tag{3.8}$$

We then have

$$u_i(y)=\sum_{j=1}^{4}\int_0^D G_{ij}(y,y')f_j(y')dy'. \tag{3.9}$$

By tracing through the definitions, we see that $\Omega_M G_{11}(y,y')$ provides the x component of magnetization $m_x(y)$ at point y, in response to an external driving field applied parallel to the

y axis, and with the spatial variation $\delta(y-y')$. Similarly $\Omega_M G_{21}(y,y')$ gives $m_y(y)$ for such a driving field, while $-\Omega_M G_{12}(y,y')$ gives $m_x(y)$, in response to a field with this spatial variation, but applied parallel to x. Finally, $-\Omega_M G_{22}(y,y')$ is $m_y(y)$ in response to a field applied parallel to x, localized at $y=y'$.

Our task is to solve Eqs. (3.7) subject to the boundary conditions that for any choice of $y', G_{3j}(y,y')$, considered a function of y, is continuous at the film surface, as is the combination $G_{4j}(y,y')+4\pi G_{2j}(y,y')$. These ensure conservation of tangential **h** and normal **b**, respectively.

It is a tedious exercise to carry through the solution of the above set of equations, but the procedure is straightforward in principle. We thus omit details, and present a tabulation of the results in the Appendix.

The Green's functions $G_{ij}(y,y')$ can be used to explore the amplitude of thermal spin fluctuations in the film, as remarked earlier. Let $m_x(\mathbf{x},t)$ and $m_y(\mathbf{x},t)$ denote the x and y components of the fluctuating magnetization. We can always Fourier transform these variables:

$$m_\alpha(\mathbf{x},t)=\int \frac{d^2k_\parallel dt}{(2\pi)^3}\, e^{i\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel}e^{-i\Omega t}m_\alpha(\mathbf{k}_\parallel\Omega;y), \quad (3.10)$$

where $m_\alpha(\mathbf{k}_\parallel\Omega;y)$ is the amplitude, within the plane $y=$const, of the thermal fluctuation of frequency Ω, and wave vector $\mathbf{k}_\parallel$, in the plane parallel to the film surfaces. If $\hbar\Omega \ll k_BT$, the limit of interest for the long-wavelength dipolar-dominated spin waves of interest here, then the fluctuation dissipation theorem tells us that $\mathrm{Im}\{G_{12}(y,y')\}$ is proportional to $(k_BT/\hbar\Omega)\langle m_x^*(\mathbf{k}_\parallel\Omega;y)m_x(\mathbf{k}_\parallel\Omega;y')\rangle_T$, where the angular brackets denote a statistical average over an ensemble at the temperature T. Similarly $\mathrm{Im}\{G_{21}(y,y')\}$ is proportional to $(k_BT/\hbar\Omega)\langle m_y^*(\mathbf{k}_\parallel\Omega;y)m_y(\mathbf{k}_\parallel\Omega;y')\rangle_T$. A precise statement of these connections is found elsewhere.[15] Again, the present paper confines its attention to wave vectors $\mathbf{k}_\parallel$ perpendicular to the magnetization.

We shall explore the thermal fluctuations sensed by a probe that extends into the sample a depth $d=\alpha_0^{-1}$, with an exponential profile. The fluctuations of wave vector $k_\parallel$ and frequency Ω parallel to the surface sample by such a probe are described by

$$s_{xx}(k_\parallel,\Omega)=\int_0^D dy\int_0^D dy' e^{-\alpha_0(y+y')}\mathrm{Im}\{G_{12}(y,y')\} \quad (3.11a)$$

while those normal to the surface are given by

$$s_{yy}(k_\parallel,\Omega)=\int_0^D dy\int_0^D dy' e^{-\alpha_0(y+y')}\mathrm{Im}\{G_{21}(y,y')\}. \quad (3.11b)$$

A description of the Brillouin light-scattering spectrum is obtained by suitably synthesizing these and other closely related spectral density functions.[5,6,15]

In Fig. 2, we show the spectral density $s_{yy}(k_\parallel,\Omega)$, calculated for the following parameters, chosen to display clearly the influence of eddy current damping. We have $\delta_0=10^{-5}$ cm for the microwave skin depth, and α_0 has been chosen equal to 10^5 cm^{-1}. The calculations are for a film whose thickness is $D=10^{-4}$ cm. Finally, $H_0=0.5$ kG, and we set $B_0=4$ kG, while we use a frequency-dependent relaxation rate $1/\tau=0.01\ \Omega$, with Ω the frequency.

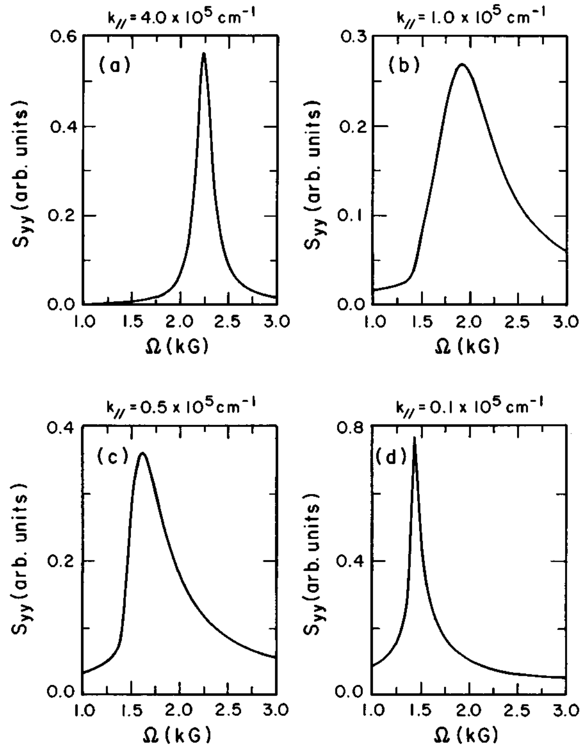

FIG. 2. The spectral density function $s_{yy}(k_\parallel,\Omega)$ defined in Eq. (3.11b), calculated for $\alpha_0=10^5$ cm^{-1}, $\delta_0=10^{-5}$ cm, $D=10^{-4}$ cm, $H_0=0.5$ kG, $H_0+4\pi M_S=4$ kG, and $1/\tau=0.01\ \Omega$, where Ω is the frequency.

These numbers do not describe any real material, but are chosen for convenience in display. Notice that one may scale the above results to apply to real materials, since the frequency dependence is controlled by the two dimensionless ratios (Ω/Ω_H) and (Ω_H/Ω_B), and we have two parameters (δ_0/D) and $k_\parallel\delta_0$ which control the wave vector dependence.

The prominent peak in Fig. 2 is the structure associated with the Damon-Eshbach wave of the film. In the absence of eddy current effects, from Eq. (2.12) we see that for $k_\parallel D\to\infty$, its frequency is $\gamma(H_0+2\pi M_s)=2.25$ kG, while as $k_\parallel D\to 0$, we have $\gamma(H_0[H_0+4\pi M_s])^{1/2}=1.41$ kG. For the largest and smallest values of $k_\parallel$ in Fig. 2, the peak indeed coincides with these limiting values.

One sees clearly from Fig. 2 the very strong eddy current damping effects when $k_\parallel\delta_0\sim 1$. There is some eddy current damping in the feature shown for $k_\parallel=4\times 10^5$ cm^{-1}, the mode becomes very broad indeed for $k_\parallel=1.0\times 10^5$ cm^{-1} and also for $k_\parallel=0.5\times 10^5$ cm^{-1}, and then narrows down when $k_\parallel$ drops to 0.1×10^5 cm^{-1}. This behavior is compatible with the behavior provided by the asymptotic formulas in Sec. II.

There are two length scales in the problem. One is the film thickness D, and the other by the microwave skin depth δ_0. When $\delta_0 \ll D$, the near surface spectral densities are controlled by the parameter $k_\parallel\delta_0$, and are insensitive to $k_\parallel D$. We may see this by calculating spectral density functions for $D=10^{-3}$ cm, a value ten times larger than that in Fig. 2. The

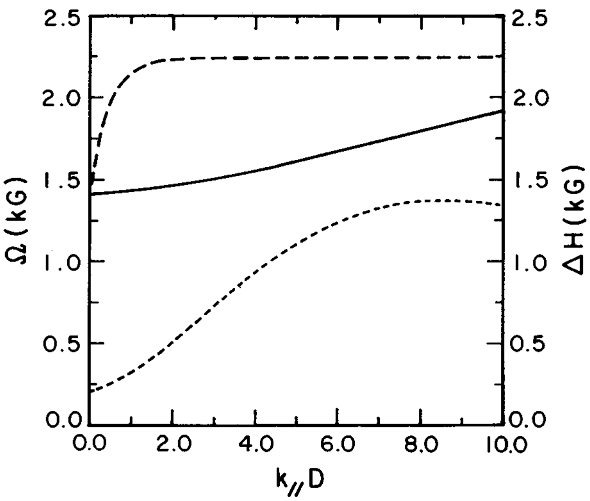

FIG. 3. From plots such as that in Fig. 2, we extract the linewidth ΔH of the Damon-Eshbach mode (dotted line), and its effective dispersion relation, shown as the solid line. The prediction of Eq. (2.12), appropriate for a film with no eddy current damping, is shown as the dashed line.

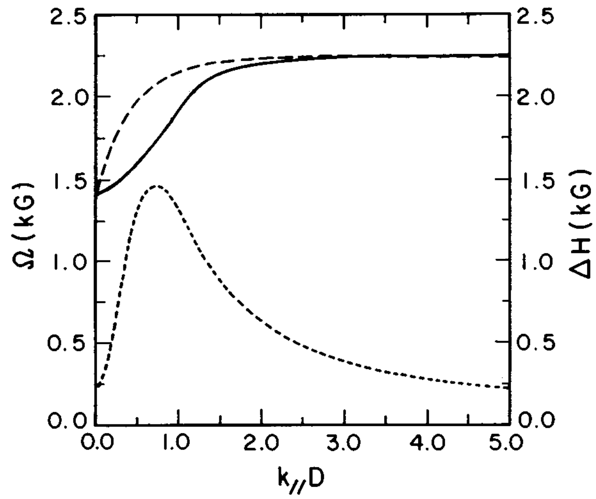

FIG. 4. The same as Fig. 3, but now the film thickness $D=10^{-5}$ cm.

results are identical to those in Fig. 2, to graphical accuracy. In essence, the microwave skin depth δ_0 acts as the active channel, to which the fields of the Damon-Eshbach wave are confined.

We may extract the variation of the linewidth with frequency, and also an effective dispersion relation, from spectral density plots such as that displayed in Fig. 2. We show this information in Fig. 3, for the model film used to generate Fig. 2. The dotted line is the frequency variation of the linewidth; the peak occurs when $k_{\parallel}\delta_0 \simeq 0.85$. The effective dispersion relation is given by the solid line, and this differs qualitatively from that appropriate to the case where eddy current damping is absent [Eq. (2.12)]. For this film, we show the prediction of Eq. (2.12) as a dotted line. Qualitatively, the solid line bears resemblance to the prediction of Eq. (2.12), but with D replaced by δ_0.

In Fig. 4 we show the variation with wavelength of the linewidth (dotted line), the effective dispersion relation (solid line) and the dispersion relation given by Eq. (2.12), for a film with thickness $D=10^{-5}$ cm. All other parameters are identical to those used in Figs. 2 and 3. We thus have a case where $D=\delta_0$. The peak in the linewidth occurs very close to $k_{\parallel}\delta_0=0.85$, the same value where we have the peak in Fig. 2. We see in this figure the dramatic fall off in the eddy current damping, as $k_{\parallel}\delta_0$ drops below unity, and also as $k_{\parallel}\delta_0$ increases above 0.85. The dispersion relation now is quite close to that applicable in the absence of eddy current damping. When $D \leqslant \delta_0$, it is the film thickness and not the skin depth which controls the effective dispersion relation.

As noted earlier, the calculations presented above use parameters chosen to illustrate eddy current damping effects, but the microwave skin depth δ_0 has been chosen equal to 10^{-5} cm, an order of magnitude smaller than the value appropriate to a transition-metal film such as Fe. The results may be applied to various materials by the appropriate scaling procedure, when one realizes the characteristic quantities enter only in the products $k_{\parallel}D$ and $k_{\parallel}\delta_0$, as remarked above.

If we have actual light-scattering studies of Fe in mind, the eddy current damping will affect the spectra severely only at values of $k_{\parallel}$ smaller than those accessed in typical experiments, which explore the regime $k_{\parallel} \sim 10^5$ cm^{-1}. We illustrate this in Fig. 5, with a series of spectra calculated for $\delta_0=10^{-4}$ cm, appropriate to Fe. Severe eddy current broadening is evident in the spectrum for which $k_{\parallel} \sim 0.1\times10^5$ cm, but its influence is rather modest at the large wave vectors. Access to the regime where $k_{\parallel} \sim 10^4$ cm^{-1} would require detection of scattered light reflected off the sample very close

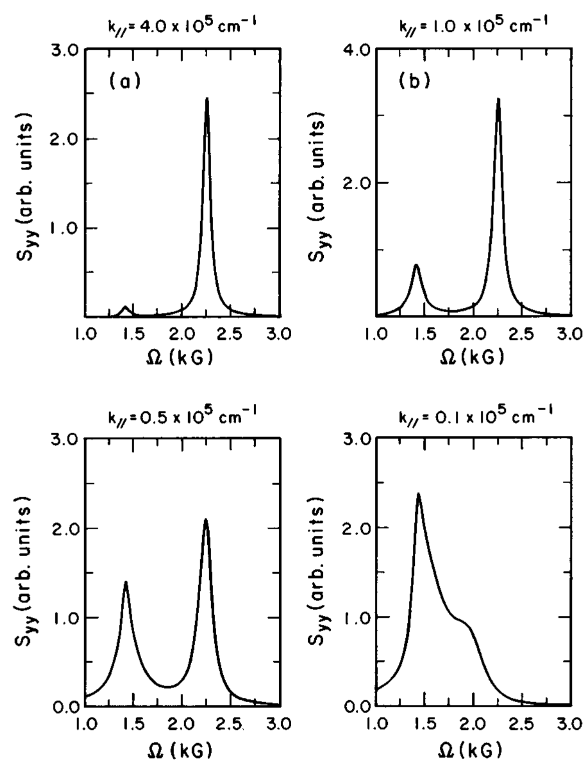

FIG. 5. The spectral density function $s_{yy}(k_{\parallel},\Omega)$ defined in Eq. (3.11b) calculated for $\alpha_0=10^5$ cm^{-1}, $\delta_0=10^{-4}$ cm, $D=5\times10^{-5}$ cm, $H_0=0.5$ kG, and $H_0+4\pi M_S=4$ kG, with also $1/\tau=0.01\ \Omega$, where Ω is the frequency.

in angle to the specular beam. This would present an experimental challenge, since surface roughness would most likely lead to an elastic component in the same angular regime, and the signals detected in such experiments are very weak.

IV. RESULTS AND DISCUSSION

We have presented the theory of eddy current damping and frequency renormalization of spin waves in conducting ferromagnetic films. We focus our attention on the Damon-Eshbach wave, when it propagates perpendicular to the magnetization. The methods used here are readily extended to other propagation geometries.

In Sec. II, we obtain an implicit dispersion relation for the waves. If $k_\parallel$ is their wave vector, we see that eddy currents influence the modes modestly in the two regimes $k_\parallel\delta_0 \ll 1$, and $k_\parallel\delta_0 \gg 1$, where δ_0 is the microwave skin depth. We give analytic approximations which apply to these limits in Eqs. (2.13), (2.14), and (2.19).

The approximate formulas, when extrapolated to the regime $k_\parallel\delta_0 \sim 1$, suggest eddy current effects can be large in this regime of wave vector. In Sec. III, we present a Green's function analysis which yields forms that may be applied to analyze the response of the film to a diverse array of probes. The fluctuation dissipation theorem allows us to use the same functions to simulate light-scattering spectra. Calculations we present indeed show eddy current damping to be strong when $k_\parallel\delta_0 \sim 1$, and we have also a dramatic renormalization of the dispersion relation, when the film thickness $D \gg \delta_0$. In essence, the skin depth acts as a channel within which the wave is trapped, and the dispersion relation becomes qualitatively similar to that of a wave confined to a film of thickness δ_0, rather than a film of thickness D.

We conclude with a discussion of the implication of these results for various experimental probes of metallic ferromagnetic films. We have in mind the case of Fe, for which the microwave skin depth $\delta_0 \sim 10^{-4}$ cm. Most of the transition metals have skin depths very close to this value.

There are two types of experiments where eddy current effects may play a role: ferromagnetic resonance studies, and Brillouin light scattering (BLS).

In an idealized ferromagnetic resonance experiment, one has $k_\parallel = 0$, if microwaves strike the film at normal incidence. We see from Eq. (2.11) that in this limit, the film responds at the ferromagnetic resonance $\Omega_H \Omega_B$, unaffected by eddy current effects. In fact, one expects a mode with nonzero $k_\parallel$ to be excited in such experiments. Suppose the sample is finite in size, possibly in the form of a square or disc with linear dimension W. Then one may expect the edges to act as pinning centers, so one will excite a mode with $k_\parallel \sim \pi/W$. For typical samples, we expect $k_\parallel\delta_0$ to be very small compared to unity under these circumstances, and eddy current effects should be quite negligible.

In the BLS studies, as noted above, $k_\parallel \sim 10^5$ cm^{-1}, so for Fe, $k_\parallel\delta_0 \sim 10$ or so. Eddy current effects are again small; we estimated in Sec. II that the linewidth of the modes may contain a contribution in the range of a few tens of Gauss. This is significant, and may affect light-scattering spectra. For example, we may expect eddy current effects in metallic magnetic superlattices to be comparable to those estimated for Fe. The additional damping introduced may obscure fine structure expected in superlattices with a finite number of layers. Such finite superlattices have a rich spectrum of spin-wave modes affected sensitively by an external magnetic field.[8] The mode structure of finite Fe/Cr (211) superlattices has been studied by BLS recently,[16] and compared with theory. The theoretical spectra are considerably richer in detail than the experimental counterparts. The difference may be due, at least in part, to broadening with origin in eddy current damping.

It is the case that Damon-Eshbach waves may be launched in ferromagnetic films via a small scale structures laid over the film. Meander lines provide an example of such an excitation source. Numerous microwave and magneto-optic devices excite modes of finite wave vector by this means.[17] If a structure used for excitation has a linear dimension w in the direction perpendicular to the direction of propagation, then the waves excited most efficiently will have $k_\parallel \sim \pi/w$. If w is in the range of 1 μm or so, a dimension quite appropriate to small scale devices, then the waves launched in an Fe film will have $k_\parallel\delta_0 \sim 1$. Eddy current damping may then have a strong effect on the pulse shape generated, and its propagation length. The Green's functions in the Appendix will allow the quantitative study of this issue. We plan to address this question in the near future.

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APPENDIX: THE EXPLICIT FORM OF THE GREEN'S FUNCTIONS

The Green's functions $G_{ij}(y,y')$ introduced in Sec. III are functions not only of y and y', but also $k_\parallel$ and Ω. In the interest of brevity, we have omitted reference to $k_\parallel$ and Ω in the main paper, and in this section as well. In what follows, we provide the form of the Green's functions only for positive values of $k_\parallel$. The frequency Ω may be either positive or negative.[13] One may generate forms valid for $k_\parallel < 0$, by noting that the functions $G_{ij}(y,y')$ are invariant under simultaneous reversal of the signs $k_\parallel$ and Ω.

In the expressions that follow, we have $\Omega_H = \gamma H_0$, $\Omega_M = \gamma M_S$, and $\Omega_B = \gamma(H_0 + 4\pi M_S)$, while Q is defined in Eq. (2.5). We take the root with Re$(Q) > 0$. Note also the definition of μ_1 and μ_2 [Eqs. (2.2)], and also that of μ_V [Eq. (2.7)]. Finally, $\tilde{\Omega} = \Omega + i/\tau$, and we suppose the film lies between $y = 0$ and $y = D$.

Each of the Green's functions may be written in the form

$$G_{ij}(y.y') = G_{ij}^{(\infty)}(y-y') + \Delta G_{ij}(y,y'), \tag{A1}$$

where $G_{ij}^{(\infty)}(y-y')$ describes the response of the infinitely extended ferromagnetic medium, and $\Delta G_{ij}(y-y')$ corrections which arise from the presence of the two film surfaces.

We have

$$G_{11}^{(\infty)}=\frac{i}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left[\tilde{\Omega}\delta(y-y')-2\pi\Omega_M\left\{k_\parallel \mathrm{sgn}(y-y')-i\,\frac{\lambda^2}{Q}\,\frac{\tilde{\Omega}\Omega_B}{\Omega_H\Omega_B-\tilde{\Omega}^2}\right\}\right]e^{-Q|y-y'|}, \tag{A2a}$$

$$G_{12}^{(\infty)}=\frac{1}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left[\Omega_B\delta(y-y')-\frac{2\pi\Omega_M}{Q}\left\{k_\parallel^2-i\,\frac{\lambda^2\Omega_B^2}{\Omega_H\Omega_B-\tilde{\Omega}^2}\right\}\right]e^{-Q|y-y'|}, \tag{A2b}$$

$$G_{21}^{(\infty)}=\frac{1}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left[\Omega_H\delta(y-y')+2\pi\Omega_M\left\{Q+i\,\frac{\lambda^2}{Q}\,\frac{\Omega_B^2}{\Omega_H\Omega_B-\tilde{\Omega}^2}\right\}\right]e^{-Q|y-y'|}, \tag{A2c}$$

and

$$G_{22}^{(\infty)}=\frac{-i}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left[\tilde{\Omega}\delta(y-y')+2\pi\Omega_M\left\{k_\parallel \mathrm{sgn}(y-y')+i\,\frac{\lambda^2}{Q}\,\frac{\tilde{\Omega}\Omega_B}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\right\}\right]e^{-Q|y-y'|}. \tag{A2d}$$

A quantity which enters the expressions for the functions $\Delta G_{ij}(y,y')$ is

$$d(\Omega,k_\parallel)=\left[[Q+k_\parallel\mu_V]^2-k_\parallel^2\left(\frac{\mu_2}{\mu_1}\right)^2\right]e^{+QD}-\left[[Q-k_\parallel\mu_V]^2-k_\parallel^2\left(\frac{\mu_2}{\mu_1}\right)\right]e^{-QD}. \tag{A3}$$

One sees from the discussion in Sec. II, and Eq. (2.10) that this quantity has a pole, for fixed $k_\parallel$, at the (complex) frequency of the Damon-Eshbach wave of the film.

Then we have the unfortunately lengthy formulas

$$\begin{aligned}\Delta G_{11}(y,y')=&\frac{i}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left(\frac{\mu_2k_\parallel+Q(\mu_1\mu_V-\mu_1)}{2\mu_1^2d(\Omega,k_\parallel)}\right)e^{-QD}\Bigg[[\mu_1Q-k_\parallel(\mu_1\mu_V-\mu_2)]\Bigg((\tilde{\Omega}+\Omega_B)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_H)k_\parallel\\&-i\,\frac{\lambda^2\tilde{\Omega}}{Q}\Bigg)e^{Q(y-y')}-[\mu_1Q+k_\parallel(\mu_1\mu_V+\mu_2)]\left((\tilde{\Omega}-\Omega_B)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}-\Omega_H)k_\parallel-i\,\frac{\lambda^2\tilde{\Omega}}{Q}\right)e^{Q(y+y')}\Bigg]-\frac{i}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\\&\times\left(\frac{\mu_2k_\parallel-Q(\mu_1\mu_V-\mu_1}{2\mu_1^2d(\Omega,k_\parallel)}\right)\Bigg[e^{-QD}[\mu_1Q-k_\parallel(\mu_1\mu_V+\mu_2)]\left((\tilde{\Omega}-\Omega_B)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}-\Omega_H)k_\parallel-i\,\frac{\lambda^2\tilde{\Omega}}{Q}\right)e^{-Q(y-y')}\\&-e^{QD}[\mu_1Q+k_\parallel(\mu_1\mu_V-\mu_2)]\left((\tilde{\Omega}+\Omega_B)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_H)k_\parallel-i\,\frac{\lambda^2\tilde{\Omega}}{Q}\right)e^{-Q(y+y')}\Bigg],\end{aligned} \tag{A4a}$$

$$\begin{aligned}\Delta G_{12}(y,y')=&\frac{1}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left(\frac{\mu_2k_\parallel+Q(\mu_1\mu_V-\mu_1)}{2\mu_1^2d(\Omega,k_\parallel)}\right)e^{-QD}\Bigg[[k_\parallel(\mu_1\mu_V-\mu_2)-\mu_1Q]\Bigg((\tilde{\Omega}+\Omega_H)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_B)k_\parallel\\&-i\,\frac{\lambda^2\Omega_B}{Q}\Bigg)e^{Q(y-y')}+[k_\parallel(\mu_1\mu_V+\mu_2)+\mu_1Q]\left((\Omega_H-\tilde{\Omega})\frac{k_\parallel^2}{Q}+(\tilde{\Omega}-\Omega_B)k_\parallel-i\,\frac{\lambda^2\Omega_B}{Q}\right)e^{Q(y+y')}\Bigg]\\&-\frac{1}{(\Omega_H\Omega_B)-\tilde{\Omega}^2)}\left(\frac{\mu_2k_\parallel-Q(\mu_1\mu_V-\mu_1)}{2\mu_1^2d(\Omega,k_\parallel)}\right)\Bigg[e^{-Qd}[k_\parallel(\mu_1\mu_V+\mu_2)-\mu_1Q]\Bigg((\Omega_H-\tilde{\Omega})\frac{k_\parallel^2}{Q}+(\tilde{\Omega}-\Omega_B)k_\parallel\\&-i\,\frac{\lambda^2\Omega_B}{Q}\Bigg)e^{-Q(y-y')}+e^{Qd}[k_\parallel(\mu_1\mu_V-\mu_2)+\mu_1Q]\left((\tilde{\Omega}+\Omega_H)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_B)k_\parallel-i\,\frac{\lambda^2\Omega_B}{Q}\right)e^{-Q(y+y')}\Bigg],\end{aligned} \tag{A4b}$$

$$\begin{aligned}\Delta G_{21}(y,y')=&\frac{1}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left(\frac{\mu_2Q+k_\parallel(\mu_1\mu_V-\mu_1)}{2\mu_1^2d(\Omega,k_\parallel)}\right)e^{-QD}\Bigg[[\mu_1Q-k_\parallel(\mu_1\mu_V-\mu_2)]\Bigg((\tilde{\Omega}+\Omega_B)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_H)k_\parallel\\&-i\,\frac{\lambda^2\tilde{\Omega}}{Q}\Bigg)e^{Q(y-y')}-[\mu_1Q+k_\parallel(\mu_1\mu_V+\mu_2)]\left((\tilde{\Omega}-\Omega_B)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}-\Omega_H)k_\parallel-i\,\frac{\lambda^2\tilde{\Omega}}{Q}\right)e^{Q(y+y')}\Bigg]+\frac{1}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\\&\times\left(\frac{\mu_2Q-k_\parallel(\mu_1\mu_V-\mu_1)}{2\mu_1^2d(\Omega,k_\parallel)}\right)\Bigg[e^{-QD}[\mu_1Q-k_\parallel(\mu_1\mu_V+\mu_2)]\left((\tilde{\Omega}-\Omega_B)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_H)k_\parallel-i\,\frac{\lambda^2\tilde{\Omega}}{Q}\right)e^{-Q(y-y')}\\&-e^{QD}[\mu_1Q+k_\parallel(\mu_1\mu_V-\mu_2)]\left((\tilde{\Omega}+\Omega_B)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_H)k_\parallel-i\,\frac{\lambda^2\tilde{\Omega}}{Q}\right)e^{-Q(y+y')}\Bigg],\end{aligned} \tag{A4c}$$

and finally

$$
\begin{aligned}
\Delta G_{22}(y,y') = {} & \frac{i}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left(\frac{\mu_2 Q + k_\parallel(\mu_1\mu_V-\mu_1)}{2\mu_1^2 d(\Omega,k_\parallel)}\right)e^{-QD}\Bigg[[\mu_1 Q - k_\parallel(\mu_1\mu_V-\mu_2)]\left((\tilde{\Omega}+\Omega_H)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_B)k_\parallel\right. \\
& \left. - i\,\frac{\lambda^2\Omega_B}{Q}\right)e^{Q(y-y')}+[\mu_1 Q + k_\parallel(\mu_1\mu_V+\mu_2)]\left((\tilde{\Omega}-\Omega_\mathrm{H})\frac{k_\parallel^2}{Q}-(\tilde{\Omega}-\Omega_B)k_\parallel + i\,\frac{\lambda^2\tilde{\Omega}_B}{Q}\right)e^{Q(y+y')}\Bigg] \\
& + \frac{i}{(\Omega_H\Omega_B-\tilde{\Omega}^2)}\left(\frac{\mu_2 Q - k_\parallel(\mu_1\mu_V-\mu_1)}{2\mu_1^2 d(\Omega,k_\parallel)}\right)\Bigg[e^{-QD}[\mu_1 Q - k_\parallel(\mu_1\mu_V+\mu_2)]\left((\Omega_H-\tilde{\Omega})\frac{k_\parallel^2}{Q}+(\tilde{\Omega}-\Omega_B)k_\parallel\right. \\
& \left. - i\,\frac{\lambda^2\tilde{\Omega}_B}{Q}\right)e^{-Q(y-y')} - e^{QD}[\mu_1 Q + k_\parallel(\mu_1\mu_V+\mu_2)]\left((\tilde{\Omega}+\Omega_H)\frac{k_\parallel^2}{Q}-(\tilde{\Omega}+\Omega_B)k_\parallel - i\,\frac{\lambda^2\Omega_B}{Q}\right)e^{-Q(y+y')}\Bigg].
\end{aligned}
\tag{A4d}
$$

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