Dynamics of interfaces in a model for molecular-beam epitaxy

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The dynamics of driven interfaces in a continuum model of growth by molecular-beam epitaxy has been studied by means of the Nozières-Gallet dynamic renormalization group technique. Relaxation of the growing film is due to both surface tension and surface diffusion. In $1+1$ dimensions, three growth regimes have been found. The first is a linearly stable state with a positive surface tension, which can be described by the Edwards-Wilkinson equation. The second is a purely diffusive state with a dynamic exponent z, different from that given by the Wolf-Villain linear theory. The last is a linearly unstable growth state in which the creation of large slopes in the interface configuration is expected. In $2+1$ dimensions, which is the critical dimension of the model, the purely diffusive regime is absent at the one loop order. However, the other two growth regimes are still present. The scaling properties of the growth states are discussed in detail.

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I. INTRODUCTION

The dynamics of vapor deposition processes has aroused a tremendous amount of interest over the past two decades [1–10], partly because of the technological importance in understanding the physical properties of the resulting thin film and partly because of the fundamental interest in the nature of kinetic roughening of the interface involved in the process. It has been found that, for a wide class of growth processes, the system self-organizes itself in some cases into a self-affine fractal morphology which belongs to a certain universality class, but in other circumstances into a growth state with an instability towards the creation of large slopes in the interface configuration. In this context, two important tasks are to identify the universality classes and to understand the origin of the dynamic scaling properties and the instabilities exhibited in these *far-from-equilibrium* growth processes.

In actual vapor deposition processes, the morphology of the interface depends sensitively on deposition conditions, among which the substrate temperature is of particular importance. When the substrate temperature is low, the mobility of the freshly landed particles is so small that the particles cannot move too far from the position where they first contact with the interface. Consequently, overhangs and vacancies are developed and the deposit is similar to an amorphous material [1–4]. In such a deposition process, growth is locally perpendicular to the existing surface and relaxation is dominated by the surface tension effect. It is generally believed that the morphology of this kind of driven interface can be described on large length scales by the Kardar-Parisi-Zhang (KPZ) equation [11]. Indeed, the KPZ equation provides a quantitative understanding of the fascinating morphology generated in a broad range of nonequilibrium processes [5–7, 9, 10].

On the other hand, many semiconducting devices or metallic multilayers are grown by molecular-beam epitaxy (MBE) at fairly high temperatures [8, 14, 16]. In such a deposition process, the mobility of the recently deposited particles is so large that the particles are able to diffuse on the surface to find the lowest energy position. Thus surface diffusion becomes the dominant relaxation process. To determine the scaling properties of these nonequilibrium processes, many analytical and numerical studies have been carried out. For example, the conserved KPZ equation was proposed as a model for these growth processes [12–15]. However, later it was realized that it is difficult to understand the physical origin of the nonlinear term appearing in this model. A more relistic approach is to study the diffusion effect on the curved interface [16–22]. Along this line, many extensive numerical simulations have been performed [19–22]. Analytically, diffusion on a curved surface was first studied by Mullins many years ago [23]. Since this original study, much work has been done and the Mullins model has been generalized to include both surface diffusion and surface tension relaxation effects for all physically interesting dimensions [16, 18, 19, 22]. Nevertheless, since this model is intrinsically nonlinear and involves many degrees of freedom, only numerical solutions have been obtained and some features are merely understood qualitatively [18].

We have performed a systematic renormalization group analysis on this generalized Mullins model for both $1+1$ and $2+1$ dimensions. We find that, in the large-distance and late-time limit, there are two growth regimes in the system for all physical dimensions. The first is linearly stable and falls into the Edwards-Wilkinson (EW) universality class [24]. The second is linearly unstable and corresponds to the large slope configurations observed in some numerical simulations [19, 20, 22]. Moreover, in $1+1$ dimensions, we also find a strong-coupling purely diffusive growth regime which is different from that described by the linear model of Wolf and Villain (WV) [25]. The purpose of this paper is to report on these calculations and describe some relevant techniques in detail. Some of the results reported here have been briefly presented in a recent paper [26].

The outline of this paper is as follows. In Sec. II,

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we introduce the generalized Mullins model and discuss some of its general properties. In Sec. III, the Nozières-Gallet dynamic renormalization group technique for systems where the fluctuation-dissipation theorem does not hold is described in detail. In Sec. IV, we present the renormalization group analysis for $1 + 1$ dimensions and the dynamic scaling properties of the fixed points are discussed. In Sec. V, we develop the renormalization group analysis for $2 + 1$ dimensions. The scaling property of the fixed point governing the linearly unstable growth regime is obtained. Finally, our conclusions and some discussions of the results are given in Sec. VI.

II. THE MODEL

It is generally believed that in molecular-beam epitaxy voids or overhangs are absent due to the fast diffusion of the particles on the surface. Therefore, the morphology of the interface can be described by a class of solid-on-solid models driven by a flux of particles. Since surface rearrangement through surface diffusion conserves volume, a continuum description of these models begins with an equation of continuity [14, 19–22, 25],

$$\frac{\partial h(\mathbf{x},t)}{\partial t} + \tilde{\boldsymbol{\nabla}}\cdot\mathbf{j}(\mathbf{x},\mathbf{t}) = v(\mathbf{x},t) + \eta(\mathbf{x},t) . \qquad (1)$$

In Eq. (1) the single-valued function $h(\mathbf{x},t)$ is the interface height variable at space-time point (x,t). The surface current density $\mathbf{j}(\mathbf{x},t)$ is tangent to the surface and the operator $\tilde{\boldsymbol{\nabla}}$ must be computed in a local coordinate system with axes parallel to the surface. The function $v(\mathbf{x},t)$ represents a deterministic vertical growth velocity. Here, we consider two kinds of contribution to $v(\mathbf{x},t)$. First, the average of the beam flux produces a constant vertical growth velocity v_0. However, this velocity disappears from the equation of motion after transformation to the comoving frame [11]. In discrete models in which evaporation is allowed there is a contribution to $v(\mathbf{x},t)$ generated by an effective surface tension. This deterministic velocity takes the form [27],

$$v(\mathbf{x},t) = -\, C_1\, \frac{\delta F[h]}{\delta h(\mathbf{x},t)} , \qquad (2)$$

where C_1 is a constant. We will include such a term in our model although it does not correspond to relaxation through surface diffusion. The results of a number of simulations [19,20,22] can only be understood if such a term is generated by the nonequilibrium nature of the growth process. One of the objectives of the present work is to investigate whether or not this term is produced under renormalization. The noise function $\eta(\mathbf{x},\mathbf{t})$ represents the fluctuation of the beam flux with zero mean value, i.e., $\langle\eta(\mathbf{x},t)\rangle = 0$ and obeying Gaussian statistics,

$$\langle\eta(\mathbf{x},t)\ \eta(\mathbf{x}',t')\rangle = 2D\ \delta^d(\mathbf{x}-\mathbf{x}')\ \delta(t-t') , \qquad (3)$$

where D is the noise spectrum strength.

An important question is how the current density $\mathbf{j}(\mathbf{x},t)$ depends on the height function $h(\mathbf{x},t)$. According to irreversible thermodynamics, the function $\mathbf{j}(\mathbf{x},t)$ is proportional to the gradient of a local chemical potential $\mu(\mathbf{x},t)$,

$$\mathbf{j}(\mathbf{x},t) \propto \tilde{\boldsymbol{\nabla}}\mu(\mathbf{x},t) . \qquad (4)$$

The local chemical potential is the amount of free energy needed to increase height $h(\mathbf{x},t)$ by a unit, i.e., $\mu(\mathbf{x},t) = \delta F[h]/\delta h$. Hence, from Eq. (4), we have [19, 22]

$$\mathbf{j}(\mathbf{x},t) = C_2\tilde{\boldsymbol{\nabla}}\mu(\mathbf{x},t) = C_2\tilde{\boldsymbol{\nabla}}\ \frac{\delta F[h]}{\delta h} , \qquad (5)$$

where C_2 is another constant.

The surface free energy functional of the system is assumed to be proportional to the d dimensional area of the interface [27],

$$F[h] = \nu \int d^dx\ \{1 + [\boldsymbol{\nabla} h(\mathbf{x},t)]^2\}^{1/2} . \qquad (6)$$

Therefore, in the continuum limit, the evolution of the interface $h(\mathbf{x},t)$ at the $d+1$- ($d \leq 2$) dimensional space-time point $(\mathbf{x},t)$, in view of Eqs. (1), (2), (5), and (6), is given by [16, 18, 19]

$$\frac{\partial h}{\partial t} = \nu\boldsymbol{\nabla}\cdot\frac{\boldsymbol{\nabla} h}{\sqrt{g(\boldsymbol{\nabla} h)}} - \kappa\sqrt{g(\boldsymbol{\nabla} h)}\ \tilde{\nabla}^2\boldsymbol{\nabla}\cdot\frac{\boldsymbol{\nabla} h}{\sqrt{g(\boldsymbol{\nabla} h)}} + \eta(\mathbf{x},t), \qquad (7)$$

where the Laplace-Beltrami operator $\tilde{\nabla}^2$ enters because the diffusion current is parallel to the interface rather than to the substrate. For $d \leq 2$, we have [19, 22, 28]

$$\tilde{\nabla}^2 = \frac{1}{\sqrt{g(\boldsymbol{\nabla} h)}}\nabla_i\left[\sqrt{g(\boldsymbol{\nabla} h)}\left(\delta_{ij} - \frac{\nabla_i h\ \nabla_j h}{g(\nabla h)}\right)\right]\nabla_j . \qquad (8)$$

Here, the function $g(x) = 1+x^2$, ∇_i stands for $\partial/\partial x_i$, and δ_{ij} is the Kronecker δ, and repeated indices imply a summation. Note that the constants C_1 and C_2 have been absorbed into the coefficients ν and κ. Clearly, Eq. (7) is invariant under the transformation $h(\mathbf{x},t) \to -h(\mathbf{x},t)$ even though the particle beam breaks the up-down symmetry. Another property of Eq. (9) is that it is considerably more complicated in $2+1$ dimensions than in $1+1$ dimensions.

Equation (7) is the generalized Mullins model which is believed to describe the dynamics of interfaces in the MBE growth process. The second term in Eq. (7), with a coefficient $\kappa > 0$, describes the surface diffusion relaxation effect, which was first derived by Mullins [23]. The first term in Eq. (7) is the surface tension term discussed above with a coefficient ν, which can be both positive and negative. The surface tension ν could be due to diffusion bias near step edges [14] and also arises if one takes into account the nonzero size of incoming particles [16]. Analytically, as will be seen below, this term must be included to obtain a consistent description of the long wavelength properties of this model.

The $1+1$-dimensional version of Eq. (7) has been studied by Golubović and Karunasiri (GK) [18]. By numerically integrating Eq. (7), GK found that the slope $\partial h/\partial x$ of the interface profile behaves like the order parameter of Ising-like systems in spinodal decomposition. Golubović and Karunasiri also argued that this model can be *approximately* transformed into an equilibrium model possessing a double-well potential, suggesting that this model would behave like Ginzburg-Landau models for second-order transitions. They attributed these striking properties of the system to the effects of the noise.

Following GK's notation of $\mathbf{m} = \boldsymbol{\nabla} h$, Eq. (7) can be rewritten as [18]

$$\frac{\partial m_i}{\partial t} = \nabla^2_{ij}\left\{\nu \frac{m_j}{\sqrt{g(m)}} - \kappa\left[\sqrt{g(m)}\left(\delta_{jk} - \frac{m_j m_k}{g(m)}\right)\right] \nabla^2_{kl}\frac{m_l}{\sqrt{g(m)}}\right\} + \nabla_i \eta(\mathbf{x},t) \ , \quad (9)$$

where the function g is given above. Note that we have an extra symmetry: $\nabla_i m_j = \nabla_j m_i$. Obviously, expanding the right-hand side of the Eq. (7) in powers of m, one has two linear terms, i.e., a Laplacian term $\nu\nabla^2 m_i$ and a Laplace square term $-\kappa\nabla^4 m_i$, arising from surface tension and surface diffusion, respectively. For positive $\nu > 0$, power counting shows that all nonlinear terms are irrelevant to the long wavelength behavior of the model. In this case, Eq. (9) reduces to the well-known Edwards-Wilkinson equation [24] and the scale-invariant solution can be easily obtained by solving this linear equation. On the other hand, when ν is negative, the whole system is linearly unstable for wave vectors q in the range $0 < q < q_c = \sqrt{-\nu/\kappa}$. In this case, both the other linear term $-\kappa\nabla^4 m_i$ and the higher order terms in the expansion become important for determining the long wavelength properties of the system. In fact, all nonlinear terms arising from surface diffusion have critical dimension $d_c = 2$ with respect to the Laplace square term $-\kappa\nabla^4 m_i$, indicating that all nonlinear terms in the power series are relevant (marginal) for $d < 2$ ($d = 2$). Therefore, the important question is as follows: What is the sign of ν in the long wavelength limit?

To answer these issues, we have applied the powerful dynamic renormalization group technique to Eq. (9). However, as mentioned above, all nonlinear terms are relevant in the physically interesting dimensions and a renormalization group analysis would seem to be a formidable task. We find that the calculation can be truncated at the one-loop order and the results turn out to be consistent. In other words, the renormalization perturbation expansion can be controlled by the powers of the leading nonlinear coupling parameters; the coefficients of higher order nonlinear terms are higher powers of the leading nonlinear couplings and can be consistently ignored to the required order. First we discuss the Nozières-Gallet dynamic renormalization group method [29], which will be used throughout this paper.

III. NOZIÈRES-GALLET RENORMALIZATION METHOD

In order to carry out a renormalization group analysis of the roughening transition, Nozières and Gallet (NG) developed an effective approach to deal with the Sine nonlinearity [29, 30]. The spirit of the NG renormalization group scheme is essentially the same as that of the classical momentum-shell technique [31, 32]. Consequently, this method is effective only to order one loop. However, it has a remarkable advantage in dealing with systems involving complicated nonlinearities. In fact, our experience shows that this renormalization scheme is the most effective and convenient one to study various nonlinear systems as long as the one loop order analysis is adequate [33].

Originally, Nozières and Gallet presented the renormalization group theory both for statics and for dynamics. However, their dynamic renormalization scheme is applicable only to systems where the fluctuation-dissipation theorem (FDT) holds. It is well known that for a dynamic system where the FDT does not hold, one must study the renormalization of the response function, the correlation function, and the vertex functions simultaneously in order to obtain the scaling properties of the system [34–37]. To study these dynamic systems, we have extented the original NG renormalization scheme to a more general form [38]. Since we believe that this technique is useful for other similar problems, we describe the general formalism of this method here in some detail. In the following two sections, we will demonstrate the application of this program to the model introduced in the previous section.

Generally, the evolution of the interface height $h(\mathbf{x},t)$ is described by the following Langevin equation

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = \mathcal{L}[h] + \mathcal{N}[h] + \eta(\mathbf{x},t) \ , \quad (10)$$

where $\mathcal{L}$ and $\mathcal{N}$ stand for linear and nonlinear terms, respectively, and the noise function η satisfies Gaussian statistics (3). The general NG renormalization group theory consists of two parts. The first is the renormalization of the equation of motion, through which all linear coefficients and nonlinear coupling constants are renormalized. In fact, the renormalization of the equation of motion is equivalent to the renormalization of the response function and the vertex functions in the usual renormalization method. The second part is the renormalization of the correlation function, from which the renormalized noise spectrum strength is obtained.

First, let us discuss the renormalization of the equation of motion. Quite generally, the interface height function $h(\mathbf{x},t)$ is a functional of noise $\eta(\mathbf{x},t)$. According to Nozières and Gallet, the coarse graining transformation for the equation of motion can be defined through the following steps.

(a) Split the noise $\eta(\mathbf{x},t)$ into two parts,

$$\eta\ (\mathbf{x},t) = \bar{\eta}\ (\mathbf{x},t) + \delta\eta\ (\mathbf{x},t) \ , \quad (11)$$

so that $\bar{\eta}(\mathbf{x},t)$ and $\delta\eta(\mathbf{x},t)$ are statistically independent.

(b) Perform a partial average over $\delta\eta$ on the height function h and define

$$\bar{h}(\mathbf{x},t) \equiv \langle h[\bar{\eta}(\mathbf{x},t) + \delta\eta(\mathbf{x},t)]\rangle_{\delta\eta} \ , \quad (12)$$

$$\delta h(\mathbf{x},t) \equiv h(\mathbf{x},t) - \bar{h}(\mathbf{x},t) \ . \quad (13)$$

Then the equations of motion for $\bar{h}$ and δh are obtained:

$$\frac{\partial \bar{h}(\mathbf{x},t)}{\partial t} = \mathcal{L}[\bar{h}] + \langle \mathcal{N}[h] \rangle_{\delta\eta} + \bar{\eta}(\mathbf{x},t), \tag{14}$$

$$\frac{\partial \delta h(\mathbf{x},t)}{\partial t} = \mathcal{L}[\delta h] + (\mathcal{N}[h] - \langle \mathcal{N}[h] \rangle_{\delta\eta}) + \delta\eta(\mathbf{x},t). \tag{15}$$

Now the original equation has been split into two equations of motion for $\bar{h}$ and δh, respectively.

(c) Solve the equation of motion (15) formally to obtain

$$\delta h(\mathbf{x},t) = \int d^d x' dt' G_0(\mathbf{x}-\mathbf{x}', t-t') \times [\delta\eta' + (\mathcal{N}[h'] - \langle \mathcal{N}[h'] \rangle_{\delta\eta})] , \tag{16}$$

where G_0 is bare response function depending on the linear term $\mathcal{L}$ and where the primes on η and h indicate that those are functions of $(\mathbf{x}', t')$. Then substitute the formal solution (16) into the equation of motion (14). As a result of nonlinear coupling terms in the dynamic equation, the parameters in the equation of motion for $\bar{h}$ are modified.

We now discuss the renormalization of the correlation function.

(a′) Formally solve the original equation of motion (10) and obtain a formal solution for $h(\mathbf{x},t)$, which has form similar to Eq. (16).

(b′) Construct a formal correlation function in the following way:

$$C(\mathbf{x},\mathbf{x}';t,t') \equiv \langle h(\mathbf{x},t) h(\mathbf{x}',t') \rangle_\eta . \tag{17}$$

Note that the average here is over η, rather than $\delta\eta$. Using the formal solution obtained in (a′), we have, explicitly,

$$C(\mathbf{x},\mathbf{x}';t,t') = \int_{'' \, '''} G_0(\mathbf{x}-\mathbf{x}'', t-t'') \times G_0(\mathbf{x}'-\mathbf{x}''', t'-t''') \, [\langle \eta''\eta''' \rangle_\eta + \langle \eta'' \mathcal{N}''' \rangle_\eta + \langle \eta''' \mathcal{N}'' \rangle_\eta + \langle \mathcal{N}'' \mathcal{N}''' \rangle_\eta] , \tag{18}$$

where we have used a convenient notation,

$$\int_{'' \, '''} = \int d^d x'' dt'' d^d x''' dt''' .$$

(c′) Substitute Eqs. (11) and (13) into the formal correlation function, then cast the correlation function in the original form. The noise spectrum strength is renormalized by calculating the constant terms in the square bracket.

There are at least two ways to understand the meaning of the above decomposition. One consists in regarding the noise $\bar{\eta}$ and $\delta\eta$ as, respectively, the long and short wavelength part of the original noise η. The other consists of the idea that both $\bar{\eta}(\mathbf{x},t)$ and $\delta\eta(\mathbf{x},t)$ have their own Fourier components, viz. $\bar{\eta}(\mathbf{k},t)$ and $\delta\eta(\mathbf{k},t)$, characterized by different spectral functions in their correlation relations. The spectral functions are finally chosen so that $\bar{\eta}(\mathbf{x},t)$ and $\delta\eta(\mathbf{x},t)$ are the long wavelength part and the short wavelength part of the noise function $\eta(\mathbf{x},t)$. Both points of view lead eventually to the same physics. In the original paper by Nozières and Gallet, the latter is used. We will choose the former here, which makes the split more intuitive. Mathematically, it can be expressed as

$$\bar{\eta}\,(\mathbf{x},t) = \int_{0<k<\Lambda'} \frac{d^2k}{(2\pi)^2} \exp\,(i\mathbf{k}\cdot\mathbf{x})\;\eta\,(\mathbf{k},t),$$

$$\delta\eta\,(\mathbf{x},t) = \int_{\Lambda'<k<\Lambda} \frac{d^2k}{(2\pi)^2} \exp\,(i\mathbf{k}\cdot\mathbf{x})\;\eta\,(\mathbf{k},t) .$$

Note that we have chosen a sharp cutoff for mathematical simplicity [39]. In actual calculation, it is convenient to set $\Lambda = 1$, $\Lambda' = \Lambda e^{-\delta l}$, δl being the width of the momentum shell. Clearly, as these expressions stand, the long wavelength part of the noise function $\bar{\eta}$ and its short wavelength counterpart $\delta\eta$ are statistically independent.

Next we proceed to discuss the renormalization group analysis of Eq. (9) using the formalism described above. Noting that the symmetry properties are different for different dimensions, the traditional ϵ-expansion method cannot be applied here and the renormalization group analyses must be performed individually for each dimension.

IV. RENORMALIZATION ANALYSIS FOR 1 + 1 DIMENSIONS

We first consider the $1+1$-dimensional case. Note that the right-hand side of Eq. (9) is not a polynomial function of the field. In order to perform the renormalization group analysis, it needs to be expanded in powers of the field m. Setting $d = 1$ and expanding the right-hand side of the Eq. (9) to the leading order nonlinear terms in m, we have

$$\frac{\partial m}{\partial t} = \nu \frac{\partial^2 m}{\partial x^2} - \kappa \frac{\partial^4 m}{\partial x^4} + \frac{\partial^2}{\partial x^2}\left[-u_0 m^3 + u_1 m^2 \frac{\partial^2 m}{\partial x^2} + u_2 m \left(\frac{\partial m}{\partial x}\right)^2 + \cdots \right] + \frac{\partial}{\partial x}\eta(x,t) , \tag{19}$$

where $(\cdots)$ indicates higher order terms and u_0, u_1, u_2 are nonlinear parameters whose bare values are $\nu/2$, 2κ, 3κ, respectively. As Eq. (19) stands, the leading nonlinear terms are cubic terms and the conserved lateral driving force proportional to $\partial^2 m^2/\partial x^2$ [12–14] does not appear. As has been pointed out previously [19], this is a consequence of the assumption that deposition does not change the nature of the diffusion process, i.e., it is still driven by energy differences and can be described using a surface Hamiltonian. If only the lowest order nonlinear terms are taken into account, the structure of Eq. (19) is quite similar to that of the Langevin equation describing dynamic critical phenomena. This similarity implies that the diffusive system will have an instability similar to a second-order transition. Therefore, a standard strategy used in the discussion of dynamic critical phenomena [31, 32] can be directly applied to study Eq. (19). Namely, regarding ν and $u_\alpha (\alpha = 0, 1, 2)$ as parameters of the same order, we determine the fixed point values ν^* and u_α^* to leading order.

A. Coarse graining of the equation of motion

Now we apply NG's approach described above to study Eq. (19). Performing the partial average over $\delta\eta$ on every term of Eq. (19), we obtain the following equation of motion for the long wavelength height field $\bar{m}(x,t)$:

$$\frac{\partial \bar{m}}{\partial t} = \nu \frac{\partial^2 \bar{m}}{\partial x^2} - \kappa \frac{\partial^4 \bar{m}}{\partial x^4} + \frac{\partial^2}{\partial x^2}\left[-u_0 \langle m^3 \rangle + u_1 \left\langle m^2 \frac{\partial^2 m}{\partial x^2} \right\rangle + u_2 \left\langle m \left(\frac{\partial m}{\partial x} \right)^2 \right\rangle + \cdots \right] + \frac{\partial}{\partial x}\bar{\eta}(x,t) \ . \quad (20)$$

Subtracting Eq. (20) from Eq. (19), we arrive at the equation of motion for the short wavelength height field δm,

$$\frac{\partial \delta m}{\partial t} = \nu \frac{\partial^2 \delta m}{\partial x^2} - \kappa \frac{\partial^4 \delta m}{\partial x^4} + \frac{\partial^2}{\partial x^2}\left\{ -u_0 (m^3 - \langle m^3 \rangle) + u_1 \left(m^2 \frac{\partial^2 m}{\partial x^2} - \left\langle m^2 \frac{\partial^2 m}{\partial x^2} \right\rangle \right) + u_2 \left[m \left(\frac{\partial m}{\partial x} \right)^2 - \left\langle m \left(\frac{\partial m}{\partial x} \right)^2 \right\rangle \right] + \cdots \right\} + \frac{\partial}{\partial x}\delta\eta(x,t) \ . \quad (21)$$

Now, the original equation of motion (19) has been split into two, one for the long wavelength part height field $\bar{m}$ and another for its short wavelength counterpart δm.

Equation (21) can be solved perturbatively in power of $u_\alpha, \alpha = 0,1,2$. Namely,

$$\delta m(x,t) = \delta m^{(0)}(x,t) + \delta m^{(1)}(x,t) + O(u^2) \ , \quad (22)$$

where

$$\delta m^{(0)}(x,t) = \int dx' dt' G_0(x - x', t - t') \frac{\partial}{\partial x'} \delta\eta(x',t') \ , \quad (23)$$

$$\delta m^{(1)}(x,t) = \int dx' dt' G_0(x - x', t - t') \frac{\partial^2}{\partial x'^2}\left\{ -3u_0 \bar{m}'^2 \delta m^{(0)'} + u_1 \left[\bar{m}'^2 \frac{\partial^2 \delta m^{(0)'}}{\partial x'^2} + 2\bar{m}' \frac{\partial^2 \bar{m}'}{\partial x'^2} \delta m'^{(0)} \right] + u_2 \left[2\bar{m}' \frac{\partial \bar{m}'}{\partial x'} \frac{\partial \delta m^{(0)'}}{\partial x'} + \left(\frac{\partial \bar{m}'}{\partial x'} \right)^2 \delta m^{(0)'} \right] \right\} \ , \quad (24)$$

where the primes on the m's and δm's mean $(\mathbf{x}', t')$. Note that in Eq. (24), we ignored constant terms and linear terms in $\bar{m}'$, which do not contribute to the one-loop order renormalization. The bare response function is given by the following equation:

$$G_0(x,t) = \theta(t) \int \frac{dk}{2\pi} \exp[ikx - (\nu + \kappa k^2)k^2 t] \ , \quad (25)$$

where $\theta(t)$ is the unit step function.

As a consequence of the nonlinear terms appearing in the equations, the two equations of motion (20) and (21) couple each other. This becomes more apparent when the field $m(x,t)$ in the nonlinear terms is written as the sum of $\bar{m}(x,t)$ and $\delta m(x,t)$ explicitly. Since the noise $\delta\eta(x,t)$ is Gaussian, $\langle \delta m(x,t) \rangle$ disappears in the equations. Accordingly, Eq. (20) can be rewritten as

$$\frac{\partial \bar{m}}{\partial t} = \nu \frac{\partial^2 \bar{m}}{\partial x^2} - \kappa \frac{\partial^4 \bar{m}}{\partial x^4} + \frac{\partial^2}{\partial x^2}\left[-u_0 \bar{m}^3 + u_1 \bar{m}^2 \frac{\partial^2 \bar{m}}{\partial x^2} + u_2 \bar{m} \left(\frac{\partial \bar{m}}{\partial x} \right)^2 + R[\bar{m}] \right] + \frac{\partial}{\partial x}\bar{\eta}(x,t) \ , \quad (26)$$

where $R[\bar{m}]$ represents coupling terms of the two equations of motion, which can be written as in the perturbation formalism,

$$R[\bar{m}] = R^{(1)}[\bar{m}] + R^{(2)}[\bar{m}] + \cdots \ , \quad (27)$$

where $R^{(1)}$ is proportional to $O(u)$, $R^{(2)}$ proportional to $O(u^2)$, and so on. Since there are three perturbation parameters in the present case, it is convenient to express $R^{(i)}$ in the following form:

$$R^{(i)}[\bar{m}] = \sum_{\alpha=0}^{2} R_\alpha^{(i)}[\bar{m}] \ , \ i = 1, 2, \ldots \ , \quad (28)$$

where the $R^{(1)}$s are given by

$$R_0^{(1)}[\bar{m}] = -3u_0 \ \bar{m} \ \langle (\delta m^{(0)})^2 \rangle \ ,$$

$$R_1^{(1)}[\bar{m}] = 2u_1 \ \bar{m} \left\langle \delta m^{(0)} \frac{\partial^2 \delta m^{(0)}}{\partial x^2} \right\rangle + u_1 \frac{\partial^2 \bar{m}}{\partial x^2} \langle (\delta m^{(0)})^2 \rangle \ ,$$

$$R_2^{(1)}[\bar{m}] = u_2 \ \bar{m} \left\langle \left(\frac{\partial \delta m^{(0)}}{\partial x} \right)^2 \right\rangle \ ,$$

and in a similar way the $R^{(2)}$ *s* can be expressed as

$$R_0^{(2)}[\bar{m}] = -6u_0 \ \bar{m} \ \langle \delta m^{(0)} \delta m^{(1)} \rangle \ ,$$

$$R_1^{(2)}[\bar{m}] = 2u_1\ \bar{m}\ \left[\left\langle \delta m^{(0)}\frac{\partial^2\delta m^{(1)}}{\partial x^2}\right\rangle + \left\langle \delta m^{(1)}\frac{\partial^2\delta m^{(0)}}{\partial x^2}\right\rangle\right] +2u_1\ \frac{\partial^2\bar{m}}{\partial x^2}\langle\delta m^{(0)}\delta m^{(1)}\rangle\ ,$$

$$R_2^{(2)}[\bar{m}] = 2u_2\ \frac{\partial\bar{m}}{\partial x}\left[\left\langle \delta m^{(0)}\frac{\partial^2\delta m^{(1)}}{\partial x^2}\right\rangle + \left\langle \delta m^{(1)}\frac{\partial^2\delta m^{(0)}}{\partial x^2}\right\rangle\right] +2u_2\bar{m}\left\langle\frac{\partial\delta m^{(0)}}{\partial x}\frac{\partial\delta m^{(1)}}{\partial x}\right\rangle ,$$

where the expressions for $\delta m^{(0)}$ and $\delta m^{(1)}$ are given in Eqs. (23) and (24), respectively, and we also ignored terms which contribute to two-loop order.

Now our task is to calculate the averages in the expressions. This can be done in three steps. First, substitute Eqs. (23) and (24) into the above expressions. Then perform the gradient expansions [31] to extract terms of the original form. Finally, carry out the momentum shell integrals, which are the coefficients of the relevant terms. These manipulations are straightforward but tedious. In Appendix A, we have listed final results for all $R^{(1)}$ and $R^{(2)}$. Here, we comment briefly on the momentum-shell integrals. Our aim is to find fixed points which are infrared stable under the dynamic renormalization group transformation in the parameter space. Physically, this stability comes from the infinite correlation length in the system. Mathematically, it is revealed in the divergence of the integrals. In the calculation, we find that some of the integrals are not divergent in the infrared limit due to the gradient operators in this model. We ignore these finite terms [40].

When these steps are performed, we have the following results:

$$R^{(1)}[\bar{m}] = -3\bar{u}_0\kappa(\delta l)\bar{m}\ +\bar{u}_1\kappa(\delta l)\frac{\partial^2\bar{m}}{\partial x^2}\ , \tag{29}$$

$$R^{(2)}[\bar{m}] = \Delta_0\ (\delta l)\bar{m}^3\ +\ \Delta_1\ (\delta l)\bar{m}^2\ \frac{\partial^2\bar{m}}{\partial x^2} +\ \Delta_2\ (\delta l)\bar{m}\left(\frac{\partial\bar{m}}{\partial x}\right)^2 , \tag{30}$$

where

$$\Delta_0 = -3u_0(3\bar{u}_0+\bar{u}_1) - 6u_1\bar{u}_0 + 3u_2\bar{u}_0\ ,$$
$$\Delta_1 = 3u_0(3\bar{u}_0 - \tfrac{1}{2}\bar{u}_1 + 3\bar{u}_2) + u_1(12\bar{u}_0+\bar{u}_1+6\bar{u}_2) +u_2(-\tfrac{3}{2}\bar{u}_0+\bar{u}_1-3\bar{u}_2)\ ,$$
$$\Delta_2 = 3u_0(3\bar{u}_0 - \tfrac{5}{2}\bar{u}_1 - 4\bar{u}_2) + u_1(9\bar{u}_0-4\bar{u}_1+8\bar{u}_2) +u_2(\tfrac{9}{2}\bar{u}_0+5\bar{u}_1-4\bar{u}_2)\ ,$$

where the reduced coupling parameters $\bar{u}_\alpha = K_1u_\alpha D/\kappa^2$ with $K_1 = 1/\pi$, and δl is the width of the momentum shell.

Substituting Eqs. (29) and (30) into Eq. (26) and rearranging the equation into the original form, we obtain the coarse grained equation of motion which has exactly the same form as Eq. (19) with the intermediate renormalized parameters ν_I, κ_I, and $u_{\alpha\ I}$, $\alpha = 0,1,2$, which are given by the following recursion relations

$$\nu_I = \nu - 3\bar{u}_0\kappa\delta l\ , \tag{31}$$

$$\kappa_I = \kappa\ -\ \bar{u}_1\kappa\delta l\ , \tag{32}$$

$$u_{\alpha\ I} = u_\alpha\ +\ \Delta_\alpha\ \delta l\ ,\ \alpha = 0\ ,\ 1\ ,2\ , \tag{33}$$

where $\Delta_\alpha, \alpha = 0,1,2,$ are given in the above equations.

B. Coarse graining of the correlation function

As mentioned above, to understand the dynamics of systems where the fluctuation-dissipation theorem does not hold, one must study the renormalization of the two-point correlation function separately. Now we discuss the renormalization of the correlation function, from which the recursion relation for the parameter D is obtained.

Following a similar manipulation as the one leading to Eq. (22), we obtain the formal solution of Eq. (19),

$$m(x,t) = \int dx'dt'G_0(x-x',t-t')\left\{\frac{\partial\eta'}{\partial x'} + \frac{\partial^2}{\partial x'^2}\left[-u_0m'^3 + u_1m'^2\frac{\partial^2m'}{\partial x'^2} +u_2m'\left(\frac{\partial m'}{\partial x'}\right)^2\right]\right\}, \tag{34}$$

where the bare response function G_0 is given in Eq. (25). The two-point correlation function in real space can be defined as follows:

$$C(x,x';t,t') = \langle m(x,t)\ m(x',t')\rangle_\eta\ . \tag{35}$$

Substituting the formal solution (34) into this definition, we arrive at the formal expression for the correlation function:

$$C(x,x';t,t') = \int_{''\ '''} G_0'(x-x'',t-t'')G_0'(x'-x''',t'-t''') \times\{\langle\eta(x'',t'')\eta(x''',t''')\rangle_\eta\ +\ f[m'',\eta'';m''',\eta'''] + f[m''',\eta''';m'',\eta'']\ \}\ , \tag{36}$$

where we have used the notation

$$\int_{''\ '''} = \int dx''dt''dx'''dt'''$$

and f is a functional of m and η, which can be expressed in leading order as,

$$f[m'',\eta'';m''',\eta'''] = \frac{\partial}{\partial x'''}\left[-u_0\langle m'''^3\eta''\rangle + u_1\left\langle \eta'' m'''^2 \frac{\partial^2 m'''}{\partial x'''}\right\rangle_\eta + u_2\left\langle \eta'' m'''\left(\frac{\partial m'''}{\partial x'''}\right)^2\right\rangle_\eta + O(u^2)\right] , \quad (37)$$

where m'' and η'' stand for $m(x'',t'')$ and $\eta(x'',t'')$, respectively. In Eq. (36), we integrated by parts and G_0' stands for $\partial G_0/\partial x$.

We now demonstrate the procedures for renormalizing the correlation function (36). Substituting the decompositions $\eta = \bar{\eta} + \delta\eta$ and $m = \bar{m} + \delta m$ into Eq. (36), we have

$$C(x,x';t,t') = \int_{''\,'''} G_0'(x-x'',t-t'')\, G_0'(x'-x''',t'-t''') \times\{\langle \eta_I(x'',t'')\eta_I(x''',t''')\rangle_\eta + f[\bar{m}'',\bar{\eta}'';\bar{m}''',\bar{\eta}'''] + f[\bar{m}''',\bar{\eta}''';\bar{m}'',\bar{\eta}'']\} , \quad (38)$$

where $f[\bar{m}'',\bar{\eta}'';\bar{m}''',\bar{\eta}''']$ is given in Eq. (37) with the replacements $m \to \bar{m}$ and $\eta \to \bar{\eta}$ and the correlation of the intermediate-stage noise function is given by

$$\langle \eta_I(x'',t'')\eta_I(x''',t''')\rangle_\eta = \langle \eta(x'',t'')\eta(x''',t''')\rangle_\eta + g(x'',t'';x''',t''') + g(x''',t''';x'',t'') , \quad (39)$$

where the function $g(x'',t'';x''',t''')$ can be expressed as follows:

$$g(x'',t'';x''',t''') = \frac{\partial}{\partial x''}\left[-u_0\langle \delta\eta'''(\delta m^{(0)''})^3\rangle + u_1\left\langle \delta\eta'''(\delta m^{(0)''})^2\frac{\partial^2 \delta m^{(0)''}}{\partial x''^2}\right\rangle + u_2\left\langle \delta\eta'''\delta m^{(0)''}\left(\frac{\partial \delta m^{(0)''}}{\partial x''}\right)^2\right\rangle\right] . \quad (40)$$

Note that we have ignored two-loop terms and irrelevant terms.

To obtain a coarse grained version of the correlation function, we must require that

$$\langle \eta_I(x'',t'')\eta_I(x''',t''')\rangle_\eta = 2D_I\, \delta(x''-x''')\delta(t''-t''') . \quad (41)$$

Equations (39) and (41) together will give the expression of D_I up to order $O(u)$. To see this clearly, we need to carry out the integrals

$$C_{0\ I}(x-x',t-t') = \int_{''\,'''} G_0'(x-x'',t-t'') \times G_0'(x'-x''',t'-t''') \times\langle \eta_I(x'',t'')\eta_I(x''',t''')_I\rangle_\eta , \quad (42)$$

using Eqs. (39) and (41), respectively. After some simple algebra, one finds that, to one-loop order, the parameter D is not renormalized, i.e.,

$$D_I = D . \quad (43)$$

This completes the coarse graining transformation for the 1 + 1-dimensional case.

C. Flow equations

The above procedures show that, to order $O(u^2)$, besides the original-type terms, no additional terms of different form are generated. This means that our original parameter space $\{\nu,\kappa,D,u_\alpha,\alpha=0,1,2\}$ is complete to this order. In other words, the higher order terms are of order $O(u^3)$ and can be consistently ignored. Using the recursion relations given in Eqs. (31)–(33), and (43), and the traditional rescaling of $x' = xe^{-\delta l}$, $t' = te^{-z\delta l}$, and $h' = he^{-\chi\delta l}$, on taking the limit of $\delta l \to 0$, we find the following flow equations:

$$\frac{d\ln u_\alpha}{dl} = z + 2\chi - 6 + 2\,\delta_{\alpha 0} + \frac{\triangle_\alpha}{u_\alpha} , \quad \alpha = 0,1,2 , \quad (44)$$

$$\frac{d\ln \nu}{dl} = z - 2 - 3\kappa\,\frac{\bar{u}_0}{\nu} , \quad (45)$$

$$\frac{d\ln \kappa}{dl} = z - 4 - \bar{u}_1 , \quad (46)$$

$$\frac{d\ln D}{dl} = z - 2\chi - 1 , \quad (47)$$

where the reduced perturbation parameters $\bar{u}_\alpha$ and the renormalization of the nonlinear couplings $\triangle_\alpha, \alpha = 0,1,2$ are given above. In Eq. (44), $\delta_{\alpha 0} = 1$ when $\alpha = 0$; otherwise, $\delta_{\alpha 0} = 0$.

From the flow equations, we have a renormalization picture to $O(u)$ for the 1 + 1-dimensional system. The nonlinear couplings, arising from both surface tension and surface diffusion, renormalize each other, which ultimately determine the fixed point value of ν^*. The parameter κ is renormalized but its fixed point value cannot be determined in the present order. This is a common feature of the renormalization of the cubic nonlinear couplings [31]. This does not affect our discussion of the linear stability of the system as long as we assume that the positive sign of the bare κ is not changed by the renormalization group transformation [41]. The noise strength D is not renormalized in the one-loop calculation. From Eq. (45), the value of surface tension at fixed point, i.e., ν^*, is proportional to $\bar{u}_0^*$. Therefore, the values of $\bar{u}_0^*$ determine the stability of the system in the long wavelength limit.

D. Fixed points and instability

To find the fixed points it is convenient to consider reduced flow equations for $\bar{u}_\alpha, \alpha = 0,1,2$. Taking derivatives with respect to l on both sides of $\ln \bar{u}_\alpha = \ln u_\alpha + \ln D - 2\ln\kappa$ and using Eqs. (44)–(47), we readily arrive at

$$\frac{d\ln \bar{u}_\alpha}{dl} = 1 + 2\delta_{\alpha 0} + 2\bar{u}_1 + \frac{\triangle_\alpha}{u_\alpha}\ ,\ \alpha = 0,1,2. \quad (48)$$

It is easy to check that there are three strong-coupling fixed points of Eq. (48): (I) $\bar{u}_0^* = 0$, $\bar{u}_1^* = \bar{u}_2^* = -\frac{1}{7}$, (II) $\bar{u}_0^* = 0.2397$, $\bar{u}_1^* = \bar{u}_2^* = -0.6447$, and (III) $\bar{u}_0^* = -1.0633$, $\bar{u}_1^* = \bar{u}_2^* = 0.8212$. The eigenvalues of these fixed points are given in the following table, from which we can analyze the stability of the fixed points.

Fixed point	λ_0	λ_{12}
I	1.85714	- 1.0000
II	-0.77649 + 2.31758 *i*	-0.77649 - 2.31758 *i*
III	-14.3675	-1.84419

The distribution of the fixed points and their flow diagram is shown schematically in Fig. 1 in the $\bar{u}_0$,u_{12} plane, where $u_{12} = (\bar{u}_1 + \bar{u}_2)/2$. The fixed point (I) is stable with respect to the Gaussian fixed point and controls the scaling behavior of purely diffusive (but nonlinear) relaxation. At this fixed point the dynamic exponent $z = 4 - \frac{1}{7} \approx 3.86$. Clearly, the growth regime governed by this fixed point is different from that described by the WV linear diffusion model [25]. The fixed point (II) with positive ν^* is in fact unstable with respect to the EW fixed point at which all nonlinear terms are irrelevant in the long wavelength limit. An interesting feature of this fixed point is that its eigenvalues are complex so that the flow lines in the parameter space are vortexlike, see Fig. 2. On the other hand at fixed point (III), one has a negative ν^*, and the system is linearly unstable. The eigenvalues at this fixed point are negative and the flow lines are shown in Fig. 3. In this circumstance, a finite system will eventually evolve into a steady state with large slopes. The precise morphology may depend on details such as boundary conditions. This fixed point is stable and has a rather large basin of attraction which is bounded by the $\bar{u}_0 = \nu = 0$ axis. At this level of approximation it therefore seems that, although the entire $\nu = 0$ axis is unstable, finite ν is *not* produced by renormalization. Furthermore, since the flow cannot cross the $\nu = 0$ axis, a negative surface tension does not become positive under renormalization nor does a positive one become negative. Note that the ratio of the two nonlinear couplings arising from the surface diffusion, $\bar{u}_1/\bar{u}_2$, has changed from the bare value 2/3 to the fixed point value 1, implying that GK's transformation [18] is *exact* at the fixed points.

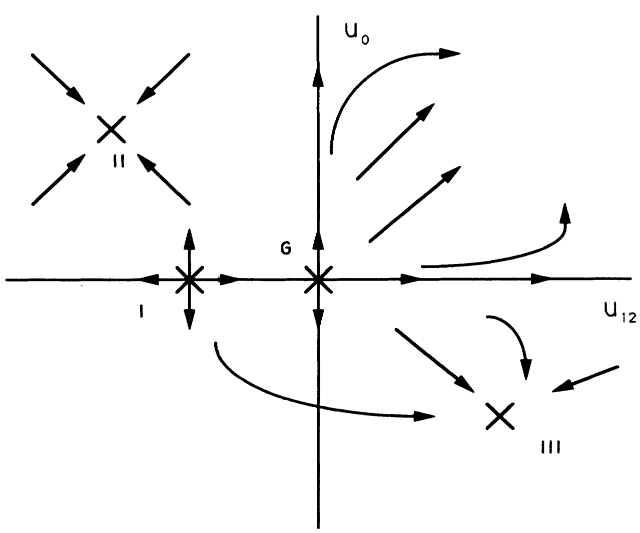

FIG. 1. Schematic renormalization group flow diagram in the $\bar{u}_0$ and u_{12} plane for $d = 1$.

V. RENORMALIZATION ANALYSIS FOR 2 + 1 DIMENSIONS

Now we turn to the of 2 + 1-dimensional case, in which the algebra is more involved than the 1 + 1-dimensional case. Setting $d = 2$ and expanding the nonlinear terms in power of m, Eq. (9) becomes

$$\frac{\partial m_i}{\partial t} = \nu\nabla^2 m_i - \kappa\nabla^4 m_i + \nabla^2_{ij}\ \mathcal{N}_j[m]\ +\ \nabla_i\eta(\mathbf{x},t)\ , \quad (49)$$

where the nonlinear terms

$$\mathcal{N}_j[m] = -u_0 m_j m^2 + u_1 m_j m_k \nabla^2 m_k + u_2 m_k m_l \nabla^2_{kl} m_j + u_3 m_k \nabla_j m_k \nabla_l m_l + u_4 m_l \nabla_l m_k \nabla_j m_k + \cdots\ . \quad (50)$$

The bare values of the nonlinear couplings are $u_0 = \nu/2$,

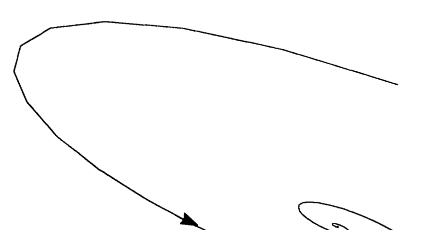

FIG. 2. The vortex flow line sinking in the fixed point II.

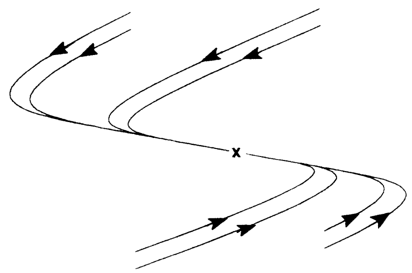

FIG. 3. The flow lines in the neighborhood of the fixed point III.

$u_1 = u_2 = u_3 = \kappa$, and $u_4 = 2\kappa$. Compared to Eq. (19), two more cubic terms appear here due to the added structural complexity. Therefore, to order $O(u)$, the dimension of the original parameter space is 8, rather than 6, as in the 1 + 1-dimensional case. However, following exactly the same method used in 1 + 1 dimension, the renormalization analysis can be done without any essential difficulty, after some rather tedious calculations.

A. Coarse graining of the equation of motion

The equation of motion (49) can be renormalized by using the scheme discussed in Sec. III. First, by splitting the noise η into two statistically independent parts, $\eta = \bar{\eta}+\delta\eta$, the original equation of motion (49) can be divided into two equations for $\bar{m}$ and δm, respectively. Namely, for $\bar{m}$ we have

$$\frac{\partial \bar{m}_i}{\partial t} = \nu\nabla^2\bar{m}_i - \kappa\nabla^4\bar{m}_i + \nabla^2_{ij}\ \langle \mathcal{N}_j[m]\rangle\ + \nabla_i\bar{\eta}(\mathbf{x},t)\ . \tag{51}$$

Subtracting Eq. (51) from Eq. (49) and defining $\delta m = m - \bar{m}$, we have

$$\frac{\partial \delta m_i}{\partial t} = \nu\nabla^2\delta m_i - \kappa\nabla^4\delta m_i + \nabla^2_{ij}\ (\mathcal{N}_j[m]\ -\ \langle \mathcal{N}_j[m]\rangle) + \nabla_i\delta\eta(\mathbf{x},t)\ , \tag{52}$$

where $\mathcal{N}_j[m]$ is given in Eq. (50). Next, we solve the equation of motion for δm formally and obtain

$$\delta m_i(\mathbf{x},t)\ = \int d^2x'dt' G_0(\mathbf{x}-\mathbf{x}',t-t')\ \{\nabla_i\delta\eta(\mathbf{x}',t') + \nabla^2_{ij}(\mathcal{N}_j[m']\ -\ \langle \mathcal{N}_j[m']\rangle)\}\ , \tag{53}$$

where the bare response function $G_0(\mathbf{x},t)$ is given by

$$G_0(\mathbf{x},t) = \theta(t)\int \frac{d^2k}{(2\pi)^2}\ \exp\ [i\mathbf{k}\cdot\mathbf{x} - (\nu+\kappa k^2)k^2t]\ . \tag{54}$$

Finally, writing $m = \bar{m} + \delta m$ in the equation of motion for $\bar{m}$ and arranging the linear terms in the original form, we may write the equation of motion for $\bar{m}$ in the form

$$\frac{\partial \bar{m}_i}{\partial t} = \nu\nabla^2\bar{m}_i - \kappa\nabla^4\bar{m}_i + \nabla^2_{ij}(\mathcal{N}_j[\bar{m}]\ +\ R_j[\bar{m}]) + \nabla_i\bar{\eta}(\mathbf{x},t)\ , \tag{55}$$

where $\mathcal{N}_j[\bar{m}]$ is given in Eq. (50) with the replacement of $m \to \bar{m}$, and $R[\bar{m}]$ represents the nonlinear coupling terms, which are given in the following equations

$$\begin{aligned}
R_{j0}[\bar{m}] &= -u_0(\bar{m}_j\langle(\delta m)^2\rangle + 2\bar{m}_i\langle\delta m_i\delta m_j\rangle)\ ,\\
R_{j1}[\bar{m}] &= u_1(\bar{m}_j\langle\delta m_k\nabla^2\delta m_k\rangle + \bar{m}_i\langle\delta m_j\nabla^2\delta m_i\rangle + \nabla^2\bar{m}_i\langle\delta m_i\delta m_j\rangle)\ ,\\
R_{j2}[\bar{m}] &= u_2(2\bar{m}_i\langle\delta m_k\nabla^2_{ik}\delta m_j\rangle + \nabla^2_{ik}\bar{m}_j\langle\delta m_i\delta m_k\rangle)\ ,\\
R_{j3}[\bar{m}] &= u_3(\bar{m}_i\langle\nabla_j\delta m_i\nabla_k\delta m_k\rangle + \nabla_k\bar{m}_k\langle\delta m_i\nabla_j\delta m_i\rangle + \nabla_j\bar{m}_i\langle\delta m_i\nabla_k\delta m_k\rangle)\ ,\\
R_{j4}[\bar{m}] &= u_4(\bar{m}_i\langle\nabla_i\delta m_k\nabla_j\delta m_k\rangle + \nabla_i\bar{m}_k\langle\delta m_i\nabla_j\delta m_k\rangle + \nabla_j\bar{m}_k\langle\delta m_i\nabla_i\delta m_k\rangle)\ .
\end{aligned}$$

The central task is to calculate these coupling terms.

Similarly to the 1 + 1-dimensional case, we first write δm in perturbation form

$$\delta m(\mathbf{x},t) = \delta m^{(0)}(\mathbf{x},t)\ +\ \delta m^{(1)}(\mathbf{x},t)\ +\ \cdots\ , \tag{56}$$

where $\delta m^{(0)}$ is proportional to $O(u^0)$, and $\delta m^{(1)}$ proportional to $O(u)$, and so on. Consequently, the R terms can be expressed perturbatively in power of u

$$R_{j\alpha}[\bar{m}] = R^{(1)}_{j\alpha}[\bar{m}]\ +\ R^{(2)}_{j\alpha}[\bar{m}]\ +\ \cdots\ , \tag{57}$$

where $R^{(1)}$ is proportional to $O(u)$, $R^{(2)}$ proportional to $O(u)$, and so on. To one-loop order, it is sufficient to calculate R to order of $R^{(2)}$. The calculation is straightforward but quite tedious. Here we only outline the main steps and make some related comments.

The first step is to obtain the expressions of $R^{(1)}$ and $R^{(2)}$ by substituting Eq. (56) into the R expressions. Next we perform the gradient expansion [31] and extract the relevant terms. At this step, one important difference from the (1 + 1)-dimensional case is that the four cubic terms with second-order derivatives are not "complete." That is, there are other terms, for instance, $m^2\nabla^2 m_j$, $m_j\nabla_k m_k\nabla_l m_l$, $m_j\nabla_k m_l\nabla_k m_l$, and so on, which do not show up in the original expansion. After the manipulation of the dynamic renormalization group, these terms are generated by the coarse graining procedure. However, a close examination reveals that they are irrelevant at the one-loop order. Finally, we carry out the integrals appearing in the coefficients of the relevant terms. Once again, the infrared-finite terms are ignored for simplicity. Taking all this into account, treating ν and $u_\alpha, \alpha = 0,...,4$ perturbatively, and including all relevant terms to leading order, we obtain the final results for the coupling terms, which are listed in Appendix B. Summing them up, we have

$$R^{(1)}_j[\bar{m}] = -2\bar{u}_0\kappa(\delta l)\bar{m}_j\ +\ \frac{1}{2}\ (\bar{u}_1\ +\ \bar{u}_2)\kappa(\delta l)\nabla^2\bar{m}_j\ , \tag{58}$$

$$R_j^{(2)}[\bar{m}] = -\Delta_0(\delta l)\bar{m}_j\bar{m}^2 + \Delta_1(\delta l)\bar{m}_j\bar{m}_k\nabla^2\bar{m}_k + \Delta_2(\delta l)\bar{m}_k\bar{m}_l\nabla^2_{kl}\bar{m}_j + \Delta_3(\delta l)\bar{m}_k\nabla_j\bar{m}_k\nabla_l\bar{m}_l + \Delta_4(\delta l)\bar{m}_l\nabla_l\bar{m}_k\nabla_j\bar{m}_k , \quad (59)$$

where the coefficients $\Delta_\alpha, \alpha = 0,1,2,3,4$, are given by the following equations

$$\Delta_0 = u_0\left(\frac{9}{2}\bar{u}_0 + \frac{9}{2}\bar{u}_1 + \frac{15}{4}\bar{u}_2 - \frac{5}{4}\bar{u}_3 - \frac{5}{4}\bar{u}_4\right) ,$$

$$\Delta_1 = u_0\left(-6\bar{u}_0 - \frac{5}{4}\bar{u}_1 - \frac{31}{4}\bar{u}_2 + \frac{19}{4}\bar{u}_3 + \frac{29}{8}\bar{u}_4\right) + u_1\left(\frac{7}{8}\bar{u}_1 - \frac{5}{8}\bar{u}_2 + 2\bar{u}_3 + \frac{15}{16}\bar{u}_4\right) + u_2\left(-2\bar{u}_2 + \frac{17}{8}\bar{u}_3 + \frac{7}{8}\bar{u}_4\right) + u_3\left(-\frac{5}{8}\bar{u}_3 + \frac{9}{16}\bar{u}_4\right) + \frac{1}{16}u_4\bar{u}_4 ,$$

$$\Delta_2 = u_0\left(-\frac{1}{2}\bar{u}_1 + 4\bar{u}_2 + \frac{1}{2}\bar{u}_3 + \frac{3}{4}\bar{u}_4\right) + u_1\left(-\frac{1}{4}\bar{u}_1 + \frac{1}{2}\bar{u}_2 + \frac{1}{2}\bar{u}_3 + \frac{5}{8}\bar{u}_4\right) + u_2\left(\frac{3}{4}\bar{u}_2 + \frac{3}{4}\bar{u}_3 + \bar{u}_4\right) + u_3\left(-\frac{1}{4}\bar{u}_3 - \frac{5}{8}\bar{u}_4\right) - \frac{3}{8}u_4\bar{u}_4 ,$$

$$\Delta_3 = u_0\left(-\bar{u}_2 + \frac{9}{2}\bar{u}_3 + \frac{1}{2}\bar{u}_4\right) + u_1\left(-\frac{1}{2}\bar{u}_2 + \frac{7}{4}\bar{u}_3\right) + u_2\left(-\bar{u}_2 + 3\bar{u}_3 + \frac{3}{8}\bar{u}_4\right) + u_3\left(-\frac{3}{4}\bar{u}_3 - \frac{5}{8}\bar{u}_4\right) + \frac{1}{8}u_4\bar{u}_4 ,$$

$$\Delta_4 = u_0\left(-2\bar{u}_0 - 2\bar{u}_1 - \frac{5}{4}\bar{u}_2 + \frac{5}{4}\bar{u}_3 + \frac{21}{4}\bar{u}_4\right) + u_1\left(-\frac{1}{2}\bar{u}_1 - \frac{7}{4}\bar{u}_2 + \frac{3}{4}\bar{u}_3 + \frac{9}{4}\bar{u}_4\right) + u_2\left(-2\bar{u}_2 + \frac{7}{4}\bar{u}_3 + \frac{15}{4}\bar{u}_4\right) + u_3\left(-\frac{1}{4}\bar{u}_3 - \bar{u}_4\right) - \frac{3}{4}u_4\bar{u}_4 ,$$

where $\bar{u}_\alpha = K_2u_\alpha D/\kappa^2$ with $K_2 = 1/2\pi$.

Substituting Eqs. (58) and (59) into Eq. (55) and casting the equation into the original form, we obtain the recursion relations for the parameters $\{\nu, \kappa, u_\alpha, \alpha = 0,1,2,3,4\}$

$$\nu_I = \nu - 2\bar{u}_0\kappa\delta l , \quad (60)$$

$$\kappa_I = \kappa - \frac{1}{2}(\bar{u}_1 + \bar{u}_2)\kappa\delta l , \quad (61)$$

$$u_{\alpha\, I} = u_\alpha + \Delta_\alpha\,\delta l , \quad \alpha = 0,1,2,3,4 , \quad (62)$$

where $\Delta_\alpha, \alpha = 0,1,...,4$, are given above.

B. Coarse graining of the correlation function

Similar to the 1 + 1-dimensional case, the correlation function has also to be renormalized separately since there is no obvious FDT in this dimension. Applying the renormalization scheme presented in Sec. III to the 2 + 1-dimensional case, after some simple algebra, we can easily obtain the recursion relation for the noise strength to one-loop order, i.e.,

$$D_I = D . \quad (63)$$

That is, as in the previous case, the noise strength D is not renormalized in the one loop order. In fact, further examination reveals that the noise strength is not renormalized in the one-loop order for cubic nonlinear couplings.

C. Flow equations and fixed points

Using the recursion relations obtained above and performing the traditional rescaling transformation, we have the following flow equations:

$$\frac{d\ln u_\alpha}{dl} = z + 2\chi - 6 + 2\delta_{\alpha 0} + \frac{\Delta_\alpha}{u_\alpha} , \quad \alpha = 0,1,2,3,4 , \quad (64)$$

$$\frac{d\ln \nu}{dl} = z - 2 - 2\kappa\frac{\bar{u}_0}{\nu} , \quad (65)$$

$$\frac{d\ln \kappa}{dl} = z - 4 - \frac{1}{2}(\bar{u}_1 + \bar{u}_2) , \quad (66)$$

$$\frac{d\ln D}{dl} = z - 2\chi - 2 , \quad (67)$$

where the renormalization of the nonlinear couplings $\Delta_\alpha, \alpha = 0,...,4$ are given above.

As for the 1 + 1-dimensional case, we consider the reduced flow equations,

$$\frac{d\ln \bar{u}_\alpha}{dl} = 2\delta_{\alpha 0} + \bar{u}_2 + \bar{u}_1 + \frac{\Delta_\alpha}{u_\alpha} , \quad \alpha = 0,1,2,3,4 . \quad (68)$$

The fixed points of Eq. (68) are given by a group of four second-order simultaneous polynomial equations. We have solved this system of simultaneous equations numerically and found one solution with negative $\bar{u}_0^*$, which is $\bar{u}_0^* = -0.1509$, $\bar{u}_1^* = -0.2530$, $\bar{u}_2^* = -0.0108$, $\bar{u}_3^* = 0.0046$, $\bar{u}_4^* = -0.1021$. There are also several other solutions with positive $\bar{u}_0^*$. As argued above, these solutions are superseded by the Edwards-Wilkinson fixed point. An unfortunate feature of the 2 + 1-dimensional case is the absence of the purely diffusive strong-coupling fixed point (I) although the Gaussian fixed point $\{\bar{u}_\alpha = 0\}$ is unstable. This is due to the fact that for $\nu = 0$ all nonlinear terms, including the ones retained in our expansion, are marginal rather than relevant at the Gaussian fixed point. To find the analog of fixed point (I) one would need to go to higher order in the perturbation expansion.

VI. DISCUSSION AND CONCLUSION

In conclusion, our dynamic renormalization group analysis of the driven interface system in the simplest continuum model for molecular-beam epitaxy shows that it can evolve into two different morphological states. The first state with a positive surface tension is linearly stable, with the long-distance, late-time behavior of the EW universality class [24]. The second state with a negative surface tension is linearly unstable and probably corresponds to the grooved state observed in numerical simulations [19, 22, 42]. However, in contrast to these discrete models, the drumhead model, driven by an external flux, does not evolve to a state with finite surface tension—if the surface tension is initially zero it remains zero under renormalization.

In 1 + 1 dimensions, we also find a purely diffusive regime with a dynamic exponent different from that described by the WV linear theory [25]. In the 2 + 1 dimensions, this purely diffusive growth regime cannot be found in the one-loop approximation due to the critical-dimension feature. We believe that, in the long wavelength and late time limit, the purely diffusive regime is one of the growth states of the system studied here. That is, for systems where both surface tension and surface diffusion are dominant, there are generally three steady growth regimes corresponding to $\nu > 0$, $\nu = 0$, and $\nu < 0$, into which the systems will evolve. Since the 2 + 1-dimensional case is at the critical dimension of the system, one must perform a two-loop analysis to find the fixed point corresponding to zero surface tension.

Methodologically, we have systematically presented the generalized Nozières-Gallet dynamic renormalization group theory. In this method, one can start the analysis in real space and the perturbation theory can be developed without graphical techniques. Our experience shows that where a one loop calculation is adequate, the NG method is the most effective and convenient renormalization group method, especially in dealing with complicated nonlinearities. Most frequently, a one-loop renormalization group analysis is sufficient to obtain the main features of a system in the long wavelength, late time limit [33]. Therefore, we believe that the formalism presented in this paper is applicable to a variety of nonlinear systems.

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APPENDIX A: RENORMALIZATION COUPLINGS FOR 1 + 1 DIMENSIONS

In this appendix we list all final results of the $R^{(1)}$'s and the $R^{(2)}$'s for 1 + 1 dimensions.

$$R_0^{(1)}[\bar{m}] = -3\bar{u}_0\ \kappa\ (\delta l)\bar{m}\ ,\ R_1^{(1)}[\bar{m}] = \bar{u}_1\ \kappa\ (\delta l)\frac{\partial^2 \bar{m}}{\partial x^2}\ ,\ R_0^{(1)}[\bar{m}]\ =\ 0\ ,$$

$$\begin{aligned} R_0^{(2)}[\bar{m}] = & -3u_0\ (3\bar{u}_0 + \bar{u}_1)\ (\delta l)\bar{m}^3 \\ & + -3u_0\ (-3\bar{u}_0 + \tfrac{1}{2}\bar{u}_1 - 3\bar{u}_2)\ (\delta l)\bar{m}^2\frac{\partial^2 \bar{m}}{\partial x^2} \\ & + -3u_0\ (-3\bar{u}_0 + \tfrac{5}{2}\bar{u}_1 - 4\bar{u}_2)\ (\delta l)\bar{m}\left(\frac{\partial \bar{m}}{\partial x}\right)^2 , \end{aligned}$$

$$\begin{aligned} R_1^{(2)}[\bar{m}] = & -6u_1\ \bar{u}_0\ (\delta l)\bar{m}^3 \\ & + u_1\ (12\bar{u}_0 + \bar{u}_1 + 6\bar{u}_2)\ (\delta l)\bar{m}^2\frac{\partial^2 \bar{m}}{\partial x^2} \\ & + u_1\ (9\bar{u}_0 - 4\bar{u}_1 + 8\bar{u}_2)\ (\delta l)\bar{m}\left(\frac{\partial \bar{m}}{\partial x}\right)^2 , \end{aligned}$$

$$\begin{aligned} R_2^{(2)}[\bar{m}] = & \ 3u_2\ \bar{u}_0\ (\delta l)\bar{m}^3 \\ & + u_2\ (-\tfrac{3}{2}\bar{u}_0 + \bar{u}_1 - 3\bar{u}_2)\ (\delta l)\bar{m}^2\frac{\partial^2 \bar{m}}{\partial x^2} \\ & + u_2\ (\tfrac{9}{2}\bar{u}_0 + 5\bar{u}_1 - 4\bar{u}_2)\ (\delta l)\bar{m}\left(\frac{\partial \bar{m}}{\partial x}\right)^2 . \end{aligned}$$

APPENDIX B: RENORMALIZATION COUPLINGS FOR 2 + 1 DIMENSIONS

In this appendix, we list all final results of the $R^{(1)}$'s and the $R^{(2)}$'s for the 2 + 1 dimensions.

$$R_{j0}^{(1)}[\bar{m}] = -2\bar{u}_0\ \kappa\ (\delta l)\ \bar{m}_j\ ,\ R_{j1}^{(1)}[\bar{m}]\ =\ \frac{1}{2}\ \bar{u}_1\ \kappa\ (\delta l)\nabla^2\ \bar{m}_j\ ,$$

$$R_{j2}^{(1)}[\bar{m}] = \frac{1}{2}\ \bar{u}_2\ \kappa\ (\delta l)\ \nabla^2\ \bar{m}_j\ ,\ R_{j3}^{(1)}[\bar{m}]\ =\ R_{j4}^{(1)}[\bar{m}]\ =\ 0\ ,$$

$$
\begin{aligned}
R^{(2)}_{j0}[\bar m] = &-u_0\left(\frac{9}{2}\bar u_0+\frac{5}{4}\bar u_1+\frac{5}{4}\bar u_2\right)(\delta l)\bar m^2\bar m_j\\
&-u_0\left(6\bar u_0-\frac{1}{4}\bar u_1+\frac{9}{4}\bar u_2-\frac{11}{4}\bar u_3-\frac{7}{8}\bar u_4\right)(\delta l)\bar m_j\bar m_r\nabla^2\bar m_r\\
&-u_0\left(\frac{1}{2}\bar u_1+\frac{1}{2}\bar u_2-\frac{1}{2}\bar u_3-\frac{3}{4}\bar u_4\right)(\delta l)\bar m_i\bar m_r\nabla^2_{ir}\bar m_j\\
&-u_0\left(2\bar u_0+\frac{5}{4}\bar u_1+\frac{11}{4}\bar u_2-\frac{1}{2}\bar u_3-\frac{3}{2}\bar u_4\right)(\delta l)\bar m_i\nabla_i\bar m_r\nabla_j\bar m_r\\
&-u_0\left(\bar u_2-\frac{3}{2}\bar u_4+\frac{1}{4}\bar u_4\right)(\delta l)\cdot\bar m_i\nabla_j\bar m_i\nabla_r\bar m_r\ ,
\end{aligned}
$$

$$
\begin{aligned}
R^{(2)}_{j1}[\bar m] = &-\frac{13}{4}u_1\ \bar u_0(\delta l)\bar m^2\bar m_j+u_1\left(-\frac{3}{2}\bar u_0+\frac{7}{8}\bar u_1-\frac{5}{8}\bar u_2+\frac{17}{8}\bar u_3+\frac{15}{16}\bar u_4\right)(\delta l)\bar m_j\bar m_r\nabla^2\bar m_r\\
&+u_1\left(-\frac{1}{4}\bar u_1-\frac{1}{4}\bar u_2+\frac{1}{4}\bar u_3+\frac{3}{8}\bar u_4\right)(\delta l)\bar m_i\bar m_r\nabla^2_{ir}\bar m_j\\
&+u_1\left(-\frac{3}{4}\bar u_0-\frac{1}{2}\bar u_1-\frac{5}{4}\bar u_2+\frac{1}{4}\bar u_3+\frac{3}{4}\bar u_4\right)(\delta l)\bar m_i\nabla_i\bar m_r\nabla_j\bar m_r\\
&+u_1\left(-\frac{1}{2}\bar u_2+\frac{3}{4}\bar u_3-\frac{1}{8}\bar u_4\right)(\delta l)\bar m_i\nabla_j\bar m_i\nabla_r\bar m_r\ ,
\end{aligned}
$$

$$
\begin{aligned}
R^{(2)}_{j2}[\bar m] = &-\frac{5}{2}u_2\ \bar u_0(\delta l)\bar m^2\bar m_j u_2\left(-\frac{11}{2}\bar u_0-2\ \bar u_2+\frac{5}{4}\bar u_3-\frac{1}{8}\bar u_4\right)(\delta l)\bar m_j\bar m_r\nabla^2\bar m_r\\
&+u_2\left(\frac{9}{2}\bar u_0+\frac{3}{4}\bar u_1+\frac{3}{4}\bar u_2+\frac{1}{2}\bar u_3+\frac{3}{4}\bar u_4\right)(\delta l)\bar m_i\bar m_r\nabla^2_{ir}\bar m_j\\
&+u_2\left(\frac{3}{2}\bar u_0-\frac{1}{2}\bar u_1-2\ \bar u_2+\frac{1}{2}\bar u_3+\frac{3}{2}\bar u_4\right)(\delta l)\bar m_i\nabla_i\bar m_r\nabla_j\bar m_r\\
&+u_2\left(-\bar u_2+\frac{3}{2}\bar u_3-\frac{1}{4}\bar u_4\right)(\delta l)\bar m_i\nabla_j\bar m_i\nabla_r\bar m_r\ ,
\end{aligned}
$$

$$
\begin{aligned}
R^{(2)}_{j3}[\bar m] = &\ \frac{5}{4}u_3\ \bar u_0(\delta l)\bar m^2\bar m_j+u_3\left(2\bar u_0-\frac{1}{8}\bar u_1+\frac{7}{8}\bar u_2-\frac{5}{8}\bar u_3+\frac{1}{16}\bar u_4\right)(\delta l)\bar m_j\bar m_r\nabla^2\bar m_r\\
&+u_3\left(\frac{1}{4}\bar u_1+\frac{1}{4}\bar u_2-\frac{1}{4}\bar u_3-\frac{3}{8}\bar u_4\right)(\delta l)\bar m_i\bar m_r\nabla^2_{ir}\bar m_j\\
&+u_3\left(\frac{3}{4}\bar u_0+\frac{1}{2}\bar u_1+\frac{5}{4}\bar u_2-\frac{1}{4}\bar u_3-\frac{3}{4}\bar u_4\right)(\delta l)\bar m_i\nabla_i\bar m_r\nabla_j\bar m_r\\
&+u_3\left(3\bar u_0+\bar u_1+\frac{3}{2}\bar u_2-\frac{3}{4}\bar u_3+\frac{1}{8}\bar u_4\right)(\delta l)\bar m_i\nabla_j\bar m_i\nabla_r\bar m_r\ ,
\end{aligned}
$$

$$
\begin{aligned}
R^{(2)}_{j4}[\bar m] = &\ \frac{5}{4}u_4\ \bar u_0(\delta l)\bar m^2\bar m_j+u_4\left(\frac{11}{4}\bar u_0+\ \bar u_2-\frac{5}{8}\bar u_3+\frac{1}{16}\bar u_4\right)(\delta l)\bar m_j\bar m_r\nabla^2\bar m_r\\
&+u_4\left(\frac{1}{4}\ \bar u_1+\frac{1}{4}\ \bar u_2-\frac{1}{4}\bar u_3-\frac{3}{8}\bar u_4\right)(\delta l)\bar m_i\bar m_r\nabla^2_{ir}\bar m_j\\
&+u_4\left(\frac{15}{4}\bar u_0+\frac{3}{2}\bar u_1+\frac{9}{4}\bar u_2-\frac{1}{4}\bar u_3-\frac{3}{4}\bar u_4\right)(\delta l)\bar m_i\nabla_i\bar m_r\nabla_j\bar m_r\\
&+u_4\left(\frac{3}{4}\bar u_0+\frac{1}{8}\bar u_1+\frac{5}{8}\bar u_2-\frac{3}{4}\bar u_3+\frac{1}{8}\bar u_4\right)(\delta l)\bar m_i\nabla_j\bar m_i\nabla_r\bar m_r\ .
\end{aligned}
$$

[1] K. L. Chopra, *Thin Film Phenomena* (McGraw-Hill, New York, 1969).
[2] D. Henderson, M. H. Brodsky, and P. Chaudhari, Appl. Phys. Lett. **25**, 641 (1974).
[3] H. J. Leamy and A. G. Dirks, J. Appl. Phys. **49**, 3430 (1978).
[4] H. J. Leamy, G. H. Gilmer, and A. G. Dirks, in *Current Topics in Materials Science*, edited by E. Kaldis (North-Holland, Amsterdam, 1980), Vol. 6.
[5] T. Vicsek, *Fractal Growth Phenomena* (World Scientific, Singapore, 1989).
[6] F. Family and T. Vicsek, *Dynamics of Fractal Surfaces* (World Scientific, Singapore, 1991).
[7] F. Family, Physica A **168**, 561 (1990).
[8] *Molecular Beam Epitaxy 1990*, edited by C. W. Tu and J. S. Harris, Jr. (North Holland, Amsterdam, 1991).
[9] *Solids Far From Equilibrium*, edited by C. Godréche (Cambridge University Press, Cambridge, 1991).
[10] *Surface Disordering: Growth, Roughening and Phase Transitions*, edited by R. Jullien, J. Kertész, P. Meakin, and D. E. Wolf (Nova Science, Commack, NY, 1992).
[11] M. Kardar, G. Parisi, and Y. C. Zhang, Phys. Rev. Lett. **56**, 889 (1986); E. Medina, T. Hwa, M. Kardar, and Y. C. Zhang, Phys. Rev. A **39**, 3053 (1989).
[12] T. Sun, H. Guo, and M. Grant, Phys. Rev. A **40**, 6763 (1989); A. Chakrabarti, J. Phys. A **23**, L919 (1990); Z. Ràcz, M. Siegert, D. Liu, and M. Plischke, Phys. Rev. A **43**, 5275 (1991).
[13] Z. W. Lai and S. Das Sarma, Phys. Rev. Lett. **66**, 2348 (1991).
[14] J. Villain, J. Phys. I (France) **1**, 19 (1991).
[15] S. Das Sarma and P. Tamborenea, Phys. Rev. Lett. **66**, 325 (1991).
[16] A. Mazor, D. J. Srolovitz, P. S. Hagan, and B. G. Bukiet, Phys. Rev. Lett. **60**, 424 (1988).
[17] S. Das Sarma and S. V. Ghaisas, Phys. Rev. Lett. **69**, 3762 (1992).
[18] L. Golubović and R. P. U. Karunasiri, Phys. Rev. Lett. **66**, 3156 (1991).
[19] M. Siegert, and M. Plischke, Phys. Rev. Lett. **68**, 2035 (1992); J. Phys. I (France) **3**, 1371 (1993).
[20] J. Krug, M. Plischke, and M. Siegert, Phys. Rev. Lett. **70**, 3271 (1993).
[21] M. Plischke, J. D. Shore, M. Schroeder, M. Siegert, and D. E. Wolf, Phys. Rev. Lett. **71**, 2509 (1993).
[22] M. Siegert and M. Plischke, Phys. Rev. E **50**, 917 (1994).
[23] W. W. Mullins, J. Appl. Phys. **28**, 333 (1957).
[24] S. F. Edwards and D. R. Wilkinson, Proc. R. Soc. London, Ser. A **381**, 17 (1982).
[25] D. E. Wolf and J. Villain, Europhys. Lett. **13**, 389 (1990).
[26] T. Sun and M. Plischke, Phys. Rev. Lett. **71**, 3174 (1993).
[27] M. Grant, Phys. Rev. B **37**, 5705 (1988).
[28] A. Maritan, F. Toigo, J. Koplik, and J. R. Banavar, Phys. Rev. Lett. **69**, 3193 (1992).
[29] P. Nozières and F. Gallet, J. Phys. (Paris) **48**, 353 (1987).
[30] F. Gallet, S. Balibar, and E. Rolley, J. Phys. (Paris) **48**, 369 (1987).
[31] S. K. Ma, *Modern Theory of Critical Phenomena* (Addison-Wesley, Reading, MA, 1976).
[32] B. I. Halperin, P. C. Hohenberg, and S. K. Ma, Phys. Rev. Lett. **29**, 1548 (1972).
[33] In some cases where the one loop calculation is not adequate, one must use field theory renormalization group method. The KPZ equation in $2+1$ dimensions is an example, see T. Sun and M. Plischke, Phys. Rev. E **49**, 5046 (1994).
[34] P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A **8**, 423 (1973).
[35] D. Forster, D. R. Nelson, and M. J. Stephen, Phys. Rev. A **16**, 732 (1977).
[36] T. Hwa, M. Kardar, and M. Paczuski, Phys. Rev. Lett. **66**, 441 (1991).
[37] Y. C. Tsai and Y. Shapir, Phys. Rev. Lett. **69**, 1773 (1992).
[38] We have discussed the dynamic roughening using the generalized NG method, see T. Sun, B. Morin, H. Guo, and M. Grant, in *Surface Disordering: Growth, Roughening and Phase Transitions* (Ref. [10]), p. 45; and (to be published).
[39] In some cases one needs a smooth cutoff, see, *Modern Theory of Critical Phenomena* (Ref. [31]), p. 215; and also see [38].
[40] If we do include these terms in the calculation (even though this complicates the algebra substantially, it still can be done in the $1+1$ case, but appears to be barely feasible in the $2+1$ case), we find that the physics is not changed. That is, no new stable fixed point is found.
[41] A negative κ corresponds to a locally unstable situation, which would be unphysical.
[42] M. Siegert and M. Plischke, Phys. Rev. Lett. **73**, 1517 (1994).