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Larmor pseudo-precession of neutron polarization at reflection

B.P. Toperverg[a,b,*], H.J. Lauter[a], V.V. Lauter-Pasyuk[a,c,d]

[a] Institut Laue Langevin, B.P. 156, Grenoble, Cedex 938042, France
[b] Petersburg Nuclear Physics Institute, 188300 Gatchina, St. Petersburg, Russia
[c] TU München, Physics Department, D-85747 Garching, Germany
[d] Joint Institute for Nuclear Research, Dubna, Moscow Region, Russia

Abstract

Classical Larmor precession (LP) of neutron polarization is considered as a result of the quantum interference between neutron spin states split in the magnetic field due to the Zeeman effect. The interference takes place if polarization is not collinear with the direction of the magnetic induction. At grazing incidence onto a magnetized film LP may occur not only in transmission through but also at reflection from the film. If the magnetic reflection potential of the film is much lower than that of the substrate then the interference between spin components of neutron wave results in anomalous LP with doubled LP phase shift. At reflection from a film whose magnetic potential is comparable with the nuclear one and with that of the substrate both spin components are totally reflected, but with a phase shift resulting in the Larmor pseudo-precession (LPP) of the neutron polarization vector. The LPP period is proportional, in contrast to LP, to the neutron wave vector component normal to the film. A Pilot experiment on a ^{57}Fe film deposited on sapphire substrate shows that one precession can be achieved for the film thickness of ~100 nm.

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Keywords: Polarized neutron reflectometry; Larmor precession

New areas of the neutron spin echo (NSE) [1] application, e.g. in reflectometry [2,3] and small angle scattering [4], has revived a considerable interest in further development of alternative ways of realization for the Larmor precession (LP) of the neutron magnetic moment $\boldsymbol{\mu}$. One of them produces [5,3] LP by radio-frequency field-induced transitions between neutron spin states split in steady field due to the Zeeman effect. The other method proposes [2] to use magnetized films instead of a homogeneous precession field traditionally created by solenoids. A physical model of the NSE spectrometer based on magnetic foils was built a while ago [6] showing the feasibility of the

*Corresponding author. Institut Laue Langevin, B.P. 156, Grenoble, Cedex 938042, France. Tel.: +33 476 20 7591; fax: +33 476 48 3906.
E-mail addresses: boris@ill.fr, boristop@thd.pnpi.spb.ru (B.P. Toperverg).

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method and also revealing its limitations [2]. The advantage apparently constitutes in the significant increase of the LP precession frequency $\omega_L = 2|\boldsymbol{\mu}||\boldsymbol{B}|/\hbar$ due to the high, up to 2 T, magnetic inductance $\boldsymbol{B} = 4\pi\boldsymbol{M}$ in ferromagnetic materials with high magnetization $\boldsymbol{M}$. A disadvantage, however, is the amount of materials in the precession path, which may cause an appreciable scattering on the way to and from the sample. Indeed, the classical precession phase $\Phi = \omega_L t$, where $t = L/v$ is the time necessary to pass the length L with the velocity v. With $\Phi \approx 463.7 B\lambda L\,\mathrm{rad\,T^{-1}\,\AA^{-1}\,cm^{-1}}$, where λ is the neutron wavelength, one needs, at normal incidence, centimeters of material to obtain a few hundreds of revolutions of the polarization vector $\boldsymbol{P}$. If, however, neutrons impinging onto the film surface at a shallow angle $\alpha \sim 10^{-2}$ then $L = d/\sin\alpha$ and the film thickness d can be reduced [6,2] to a few dozens of micrometers, or even less for long-wavelength neutrons.

On the other hand, at sufficiently low angles of incidence classical description of LP fails. This is due to the fact, that it neglects a variation of the interaction potential over a scale comparable with the radiation wavelength λ [7]. At small α the normal to the surface component $p_0 = k\sin\alpha$ of the neutron wave vector $\boldsymbol{k}$ becomes small, while the corresponding projection $\lambda_\perp = 2\pi/p_0 = \lambda/\sin\alpha$ of the wavelength becomes larger than the interfacial region of the film. The limitation for the classical description becomes obvious after the quantum mechanical derivation provides a result comprising the classical result as a limiting case. Such derivation is based on the fact that spin interaction with magnetic field is described by the Zeeman term $\hat{\mathscr{H}} = -\hat{\boldsymbol{\mu}}\boldsymbol{B}$, where $\hat{\boldsymbol{\mu}} = \mu\hat{\boldsymbol{\sigma}}$, and $\hat{\boldsymbol{\sigma}}$ is the vector of the Pauli matrices. The 2×2 matrix $\hat{\mathscr{H}}$ has eigenvalues $\pm\mu B$ and is diagonal in the representation with the quantization axis along the field. Then the Zeeman splitting between the neutron spin states is equal to $\hbar\omega_L$ corresponding to the LP frequency. Transitions between the energy levels require an energy exchange between the neutron and alternating field [5] which induces those transitions.

In the case of the time-independent field addressed hereinafter, neutron energy is conserved and no transitions between the Zeeman levels are possible. Then LP is a purely elastic phenomenon related to the interference between spin states split in the static field [8]. If initially only one spin state in ambient guide (infinitesimally small) field is populated, i.e. the beam is perfectly polarized, then entering the field $\boldsymbol{B}$ non-collinear with the initial quantization axis both spin states with either positive and negative spin projections onto the direction $\boldsymbol{b} = \boldsymbol{B}/|\boldsymbol{B}|$ are populated. This, in particular, happens at incidence onto the magnetized film with a relatively sharp and flat interface. Due to the dependence of $\boldsymbol{B}(z)$ on the coordinate z normal to the surface the translational invariance in this direction is violated and the normal component of the neutron wave vector is not conserved. Its fraction transferred to the field depends on the neutron spin state. Correspondingly, inside the range of homogeneous field each of neutron spin components propagates with its own phase velocity $v_\pm = \hbar k_\pm/m$, where $k_\pm = |\boldsymbol{k}_\pm|$ are the wave numbers determined by the eigenvalues $\hbar k_\pm = \sqrt{2m(E \mp \mu B)}$ of the stationary Schrödinger equation, $E = \hbar^2 k^2/2m$ is the neutron energy, and $k = |\boldsymbol{k}|$ is its wave number in the field free space. If the film surface is flat over a distance greater than the lateral projection of the coherence length of the incident beam, then the wave vector projection $\boldsymbol{\kappa}_i$ onto the surface is also conserved. The solution of the Schrödinger equation $|\Psi(\boldsymbol{r})\rangle = \exp(\mathrm{i}\boldsymbol{\kappa}\boldsymbol{\rho})|\psi(z)\rangle$ in this case is factorized into a product of functions depending either on the lateral coordinate $\boldsymbol{\rho}$, or on the transverse coordinate z. The vector of states $|\psi(z)\rangle = |\psi(p, z)\rangle$ depends on the wave number p. In the field-free space $p = p_0 = k\sin\alpha$, while within the field range the degeneracy over spin states is lifted and $p = p_\pm = \sqrt{p_0^2 - p_{\mathrm{c}\pm}^2}$, where $p_{\mathrm{c}\pm}^2 = p_{\mathrm{N}}^2 \pm p_{\mathrm{M}}^2$ are the critical wave numbers for one or the other spin components, p_{N} is the critical wave number of the total reflection from nuclear, while p_{M} from magnetic part of the optical potential, and $\hbar^2 p_{\mathrm{c}\pm}^2/2m = \hbar\omega_L/2$. Correspondingly, each component of the 2D vector $|\psi(z)\rangle$ of spin states inside the homogeneous field range is, in general, a superposition of the eigenfunctions $\psi_\pm(z) = \psi(p_\pm, z)$ of the Schrödinger equation.

In the next section we show that interference of those eigenfunctions results in LP in the classical limit, but is distorted approaching the range of the total reflection.

In Section 2 we demonstrate that below the total reflection the polarization precession can be of two types. One is formed if the film magnetization and the Zeeman splitting are small, while both spin components are totally reflected from the non-magnetic substrate. More interesting is the interference between one spin component reflected from the substrate and that is totally reflected from the front face of the film. This interference results in pseudo-precession with the frequency matched to the film thickness and, in contrast to the classical LP, is proportional to neutron velocity.

In the concluding section we bring experimental evidence for the pseudo-precession measured at reflection from single crystalline ^{57}Fe film magnetized at 90° with respect to the incident polarization. It is shown that 100 nm thick film is sufficient to obtain at least one full precession of the polarization at $\lambda = 4.4$ Å within the total reflection plateau.

1. Larmor precession in transmission

The evolution of the polarization vector $\boldsymbol{P}$ travelling with a velocity v a distance L through homogeneous field is described by the equation

$$P^{\alpha} = \Omega^{\alpha\beta} P_i^{\beta},$$
$$\Omega^{\alpha\beta} = b^{\alpha}b^{\beta} + (\delta^{\alpha\beta} - b^{\alpha}b^{\beta})\cos\Phi - \varepsilon^{\alpha\beta\gamma}b^{\gamma}\sin\Phi, \quad (1)$$

where $\Omega^{\alpha\beta}$ is 3×3 matrix of rotation around the direction determined by the unit vector $\boldsymbol{b} = \boldsymbol{B}/|\boldsymbol{B}|$ for the angle Φ, and α, β, γ denote the Cartesian indices x, y or z. The matrix $\Omega^{\alpha\beta}$ in Eq. (1) is represented as a sum of three orthogonal terms. The first one transforms the projection of the vector $\boldsymbol{P}_i$ onto the field into the projection of $\boldsymbol{P}$ onto the same direction. This projection is just conserved in homogeneous field. The second, diagonal, and the third, off-diagonal, matrices describe transformation of the projections within the plane perpendicular to $\boldsymbol{b}$ accordingly to the rotation of polarization vector rotation around vector $\boldsymbol{b}$.

In the vector form Eq. (1) reads:

$$\boldsymbol{P} = \boldsymbol{b}(\boldsymbol{b}\boldsymbol{P}_i) + [\boldsymbol{P}_i - \boldsymbol{b}(\boldsymbol{b}\boldsymbol{P}_i)]\cos\Phi + [\boldsymbol{b} \times \boldsymbol{P}_i]\sin\Phi$$

and is usually considered as a solution of either of the Bloch equation $\mathrm{d}\boldsymbol{P}/\mathrm{d}t = -\gamma[\boldsymbol{B} \times \boldsymbol{P}]$ or of the time-dependent Schrödinger equation for the neutron spin operator. However, if the field does not vary in time it is more natural to obtain LP just solving the stationary Schrödinger equation. This is easily done by taking into account that at transmission through the field the initial 2D vector $|\psi_i\rangle$ of neutron spin states is transformed into the final vector of states $|\psi\rangle = \hat{T}|\psi_i\rangle$ by the 2×2 transmittance matrix $\hat{T}$. Then the polarization vector $\boldsymbol{P}$ is determined by the equation:

$$2\boldsymbol{P} = \overline{\langle\psi_i|\hat{\boldsymbol{\sigma}}|\psi_i\rangle}/\overline{\langle\psi_i|\psi_i\rangle} = \mathrm{Tr}\{\hat{\rho}_i\hat{T}^{+}\hat{\boldsymbol{\sigma}}\hat{T}\}/\mathrm{Tr}\{\hat{\rho}_i\hat{T}^{+}\hat{T}\},$$

where

$$\hat{\rho}_i = \overline{|\psi_i\rangle\langle\psi_i|} = \{1 + (\boldsymbol{P}_i\boldsymbol{\sigma})\}/2$$

is the spin density matrix of incoming neutron states, and $\hat{\boldsymbol{\sigma}}$ is the vector of Pauli matrices.

The polarization vector $\boldsymbol{P}$ is a rather convenient object for theoretical consideration, but certainly not directly accessible experimentally. Instead, spin–flip and non-spin–flip neutron intensities are measured at that or another configuration of polarizer and analyzer. The latter one can easily be included into description above just by introduction its density matrix

$$\hat{\rho}_f = \overline{|\psi_f\rangle\langle\psi_f|} = \{1 + (\boldsymbol{P}_f\boldsymbol{\sigma})\}/2,$$

where the vector $\boldsymbol{P}_f$ plays a role of the spin analyzer efficiency along the direction this vector points to. Then the transmission is determined as

$$\mathscr{T} = \overline{\langle\psi_f|T|\psi_i\rangle^{*}\langle\psi_f|T|\psi_i\rangle} = \mathrm{Tr}\{\hat{\rho}_i\hat{T}^{+}\hat{\rho}_f\hat{T}\}.$$

The matrix of the transmission amplitude through a region of homogeneous field can be generally represented as

$$\hat{T} = \tfrac{1}{2}\{[T_+ + T_-] + (\boldsymbol{b}\hat{\boldsymbol{\sigma}})[T_+ - T_-]\},$$

where $T_{\pm} = |T_{\pm}|\mathrm{e}^{\mathrm{i}\chi_{\pm}}$ are (complex) eigenvalues of the matrix $\hat{T}$, and $\chi_{\pm}$ are corresponding phases. This representation immediately yields the following set equations for the projections of the polarization vector and for the transmission coefficient:

$$P^{\alpha} = [\Omega_T^{\alpha\beta} P_i^{\beta} + b^{\alpha}\mathscr{P}_T]/[\mathscr{T}_0 + \mathscr{P}_T(\boldsymbol{bP}_i)], \quad (2)$$

$$\mathscr{T} = \tfrac{1}{2}\{\mathscr{T}_0 + \mathscr{P}_0 b^{\alpha}(P_i^{\alpha} + P_f^{\alpha}) + \Omega_T^{\alpha\beta} P_i^{\alpha} P_f^{\beta}\}, \quad (3)$$

where

$$\mathscr{P}_T = [|T_+|^2 - |T_-|^2]/2,$$
$$\mathscr{T}_0 = [|T_+|^2 + |T_-|^2]/2.$$

Eq. (2) is similar to Eq. (1), but contains an extra term $\boldsymbol{P}_T = \boldsymbol{b}\mathscr{P}_T$. It has a clear quantum origin and is due to the difference $\mathscr{P}_T$ in transmissions for different spin components through the field range. It, actually, accounts for the difference in reflections for those components, i.e. the phenomenon totally ignored in the classical approach. The sum $\mathscr{T}_0$ determines the transmission through the field range of unpolarized neutrons, while denominator in Eq. (2) determines the transmission if they are initially polarized. The "rotation" matrix $\Omega_T^{\alpha\beta}$ in Eqs. (2) and (3) is represented similar to Eq. (1) and contains three orthogonal terms:

$$\Omega_T^{\alpha\beta} = \{b^{\alpha}b^{\beta}\mathscr{T}_0 + [(\delta^{\alpha\beta} - b^{\alpha}b^{\beta})\cos\Phi_T - \varepsilon^{\alpha\beta\gamma}b^{\gamma}\sin\Phi_T]|T_+T_-|\}, \quad (4)$$

where $\Phi_T = (\chi_+ - \chi_-)$.

This equation reduces to Eq. (1) at $|T_+| = |T_-| = 1$ and $\Phi_T = \Phi$, i.e. only far away from the total reflection edge, where

$$p_0 \ll p_{M\pm}, p_{i\pm} \approx p_0 - p_{M\pm}^2/2p_0$$

and $(\chi_+ - \chi_-) \approx -\omega_L(\mathrm{d}/v_{0i})$. Then $\mathscr{T} \approx 1$, $\boldsymbol{P}_T \approx 0$ and Eq. (2) collapses to Eq. (1). Approaching the critical edge one should take into account distortions of the LP which are due to optical effects. One of them is refraction which influences the phase velocity at transmission through the field range and distorts the LP phase. The other is reflection, which reduces the neutron beam flux transmitted through this range. This reduction depends on the neutron spin projection onto the field direction. In the limiting case one spin component can be totally reflected at $|T_-| = 1$, while the other component can be totally transmitted at $|T_+| = 0$. Then $\Omega_T^{\alpha\beta} = b^{\alpha}b^{\beta}/[1 - (\boldsymbol{bP}_i)]$, and the transmitted beam is totally polarized inthe direction opposite to the field, $\boldsymbol{P} = -\boldsymbol{b}$, independently of the incident polarization. This trivial example shows that if the optical effects are important then the consists of the neutron spin interaction with the magnetic field consists not only in LP of polarization.

If the magnetic film is deposited onto a non-magnetic substrate then transmittances, $T_{\pm}$, for corresponding spin components are written explicitly as

$$T_{\pm} = \frac{T_{1\pm}T_{s\pm}e^{i\varphi_{\pm}}}{1 + R_{1\pm}R_{s\pm}e^{2i\varphi_{\pm}}} = |T_{\pm}|e^{i\chi_{T\pm}}, \quad (5)$$

with $T_{1\pm} = 2p_0/(p_0 + p_{\pm})$ and $T_{s\pm} = 2p_{\pm}/(p_{\pm} + p_s)$ having the form of Fresnel transmission amplitudes through the front or, respectively, back faces of the film. Similarly, $R_{1\pm} = (p_0 - p_{\pm})/(p_0 + p_{\pm})$ and $R_{s\pm} = (p_{\pm} - p_s)/(p_0 + p_{\pm})$ are corresponding Fresnel amplitudes of reflection. The phase shifts $\varphi_{\pm} = p_{\pm}d$ are determined by the wave numbers $p_{\pm} = \sqrt{p_0^2 - p_N^2 \mp p_M^2}$, with p_N and p_M being the critical wave numbers of the total reflection from the nuclear and magnetic optical potential, and $p_s = \sqrt{p_0^2 - p_c^2}$, where p_c is the critical wave number for the substrate.

From Eq. (5) it follows that, as expected, the classical result is valid far away of the critical edges of the total reflections. Otherwise LP receives appreciable amplitude and frequency quantum distortions. This illustrates Fig. 1, where spin–flip and non-spin–flip intensities transmitted through and reflected from the magnetic film are plotted as functions of the incident wave vector. For the sake of simplicity no nuclear optical potential contribution from either film, or substrate is assumed. Nonetheless, one can see that the signal strongly deviates from harmonicity not only in the vicinity of the critical edge $p_0 = p_M \approx 0.004\,\text{Å}^{-1}$ for the positive spin component, but is also distorted well above the p_M. On the other hand, oscillations at the incidence below the critical edge are clearly visible in the reflection channel.

2. Anomalous and pseudo-precession

In the vast majority of experiments on polarized neutron reflectometry (PNR) the initial neutron

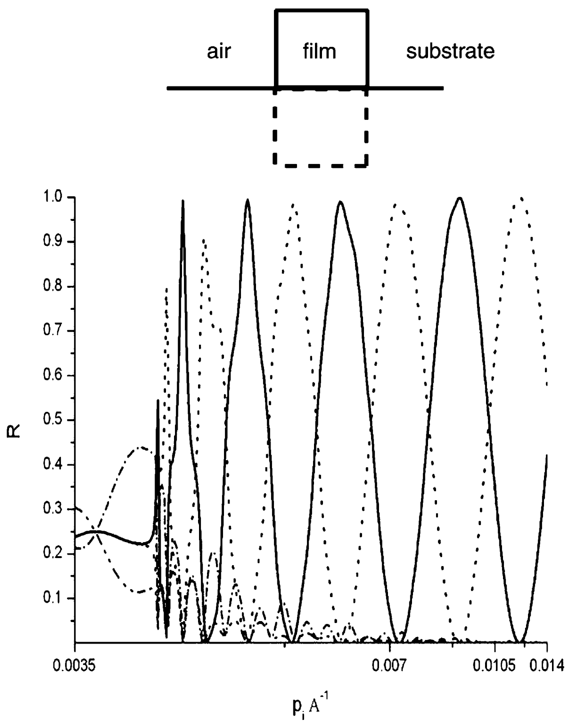

Fig. 1. Oscillations in spin–flip (solid line) and non-spin–flip(-dashed line) intensities transmitted through and reflected from the film magnetized perpendicular to the neutron polarization vector. Calculations produced for the film thickness $d = 0.5\mu$, with magnetic inductance $B = 1$ T corresponding to the critical wave number $p_c \approx 0.004\,\text{Å}^{-1}$. Nuclear optical potentials of the film and substrate are ignored for clarity. Wave numbers are indicated in reciprocal scale.

polarization vector $\boldsymbol{P}_i$ is directed collinear with the vector of the film magnetization $\boldsymbol{M}$. Then due to the Zeeman splitting in the magnetic field $\boldsymbol{B} = 4\pi\boldsymbol{M}$ one can observe two critical edges of the total reflection by flipping of the polarization direction with respect to $\boldsymbol{M}$. The effect is easily observed if the optical potentials for both positive and negative neutron spin projections are greater than the optical potential of the substrate, and if the film is thick enough.

If the vectors $\boldsymbol{P}_i$ and $\boldsymbol{M}$ are not collinear then reflectivity may show more complicated behavior related, in particular, to the rotation of the polarization vector $\boldsymbol{P}$ of reflected neutrons around the vector $\boldsymbol{b}$. Representing the reflectance matrix as $\hat{R} = \frac{1}{2}\{[R_+ + R_-] + (\boldsymbol{b}\hat{\boldsymbol{\sigma}})[R_+ - R_-]\}$, where $R_\pm$ are eigenvalues of $\hat{R}$, one obtains the set of equations for the vector $\boldsymbol{P}$ at reflection and reflectivity $\mathscr{R}$:

$$P^\alpha = [\Omega_R^{\alpha\beta} P_i^\beta + b^\alpha \mathscr{P}_R]/[\mathscr{R}_0 + \mathscr{P}_R(\boldsymbol{b}\boldsymbol{P}_i)], \tag{6}$$

$$\mathscr{R} = \tfrac{1}{2}\{\mathscr{R}_0 + \mathscr{P}_R b^\alpha (P_i^\alpha + P_f^\alpha) + \Omega_R^{\alpha\beta} P_i^\alpha P_f^\beta\}, \tag{7}$$

where $\mathscr{P}_R = [|R_+|^2 - |R_-|^2]/2$, and $\mathscr{R}_0 = [|R_+|^2 + |R_-|^2/]2$. Equations for the rotation matrix $\Omega_R^{\alpha\beta}$ follow from Eqs. (3) by the substitution $T_\pm$ for $R_\pm = |R_\pm| e^{i\chi_{R\pm}}$ and Φ_T for $\Phi_R = (\chi_{R+} - \chi_{R-})$, respectively.

The reflection amplitudes for a single magnetic film deposited onto a non-magnetic substrate have the form similar to Eq. (5).

$$R_\pm = \frac{R_{1\pm} + R_{s\pm} e^{2i\varphi_\pm}}{1 + R_{1\pm} R_{s\pm} e^{2i\varphi_\pm}}. \tag{8}$$

Note, that for a "transparent" substrate $R_{1\pm} = -R_{s\pm}$. Then for the case of totally compensated nuclear optical potential considered in the previous section there is a narrow range where spin–flip and non-spin–flip reflectivities oscillate in almost anti-phase, as seen in Fig. 1. However, the amplitude of those oscillations is rather low due to low reflectivity for negative spin projection, which has no total reflection region.

If, however, the optical potential of the substrate is high, while magnetization of the film with zero nuclear reflection potential is low, then $|R_{1\pm}| \ll 1$ in a broad range below p_s, and $R_\pm \approx e^{i(\chi_{s\pm} + 2\varphi_\pm)}$. Fig. 2 shows nice oscillations due to the anomalous precession of the polarization at total reflection from the substrate. The frequency of this precession is $\omega = 4\,\mu B/\hbar$ doubled with respect to the normal LP. This doubles the precession angle $\Phi_R = 2\omega_L L/v$ gained in reflection which in terms of classical approach can be explained as a result of sum of the neutron pathes to and from the substrate through the range of the field. The amplitude of the oscillations is close to 100%, but their line shape becomes distorted at higher field due to reflection from the magnetic potential. Therefore, in order to obtain an appreciable number of oscillations below p_c, but still above p_M, one needs a rather thick film. In Fig. 2 the thickness is chosen as $d = 100\mu$ which provides 5 oscillation at $B = 10$ mT within the range $0.002 \leqslant p_0 \leqslant 0.005\,\text{Å}^{-1}$. Beyond this range the precession signal becomes appreciably distorted.

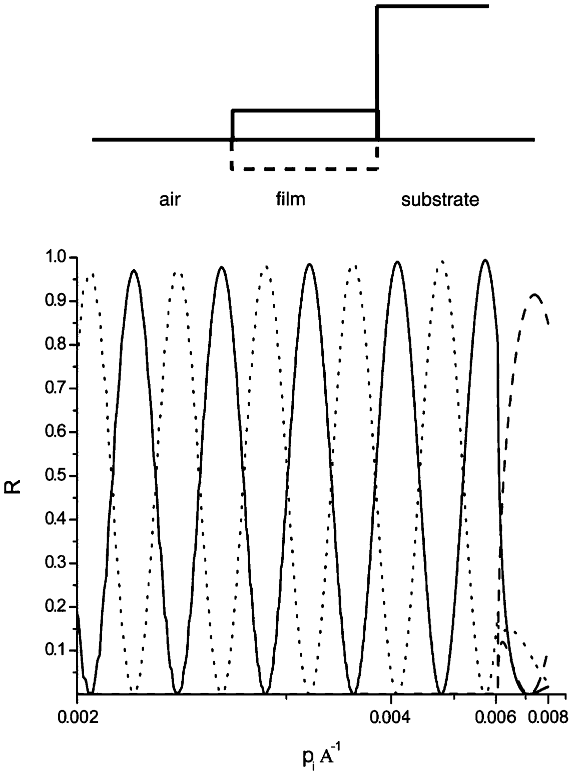

Fig. 2. Oscillations in spin–flip and non-spin–flip intensities reflected from the film magnetized perpendicular to the neutron polarization vector. Calculations produced for the film thickness $d = 100\mu$, with magnetic inductance $B = 0.01\,\mathrm{T}$. Nuclear optical potentials of the film is zero, substrate is sapphire.

More attractive is the case [9] when the film is deposited on the substrate which nuclear optical potential is close to the potential of the film for the neutron spin projection onto ***b***, while the reflection potential for the other spin projection is close to zero. If $p_- \approx p_0$, $p_+ \approx p_s$, then from Eq. (8) it follows that $R_- \approx e^{\chi_s}$, and $R_- \approx e^{i(2\varphi_0+\chi_s)}$, where $\varphi_0 = p_0 d$. This means that the phase difference $(\chi_{R+} - \chi_{R-}) = 2\varphi_0$ and the precession angle $\Phi_i = 4\pi \sin \alpha_i (d/\lambda)$ is in contrast to the conventional LP, directly proportional to the wave vector, or the neutron velocity. The period of oscillations of the polarization projection is just matched to the film thickness.

Fig. 3 shows the result of calculations carried out for a ^{57}Fe, film deposited onto a sapphire substrate with $p_s \approx 8.46 \times 10^{-3}\,\mathrm{\mathring{A}}^{-1}$. The latter is close, but not exactly matched to the critical wave number $p_+ \approx 9.35 \times 10^{-3}\,\mathrm{\mathring{A}}^{-1}$ of the total reflection from ^{57}Fe of the positive spin component. Moreover, there is no critical edge for the total reflection for the negative spin projection and the corresponding critical wave number $p_- \approx i6.13 \times 10^{-3}\,\mathrm{\mathring{A}}^{-1}$ is imaginary. However, Fig. 3 shows that within the range between $0.003 \leqslant p_0 \leqslant 0.007$ oscillation in the reflectivity curves are well harmonic.

3. Experiment

A neutron reflection experiment was performed on an iron film grown with molecular beam epitaxy on a sapphire substrate and covered with a thin Cr layer $Al_2O_3/^{57}Fe(966\,\mathrm{\mathring{A}})/Cr(22\,\mathrm{\mathring{A}})$. A

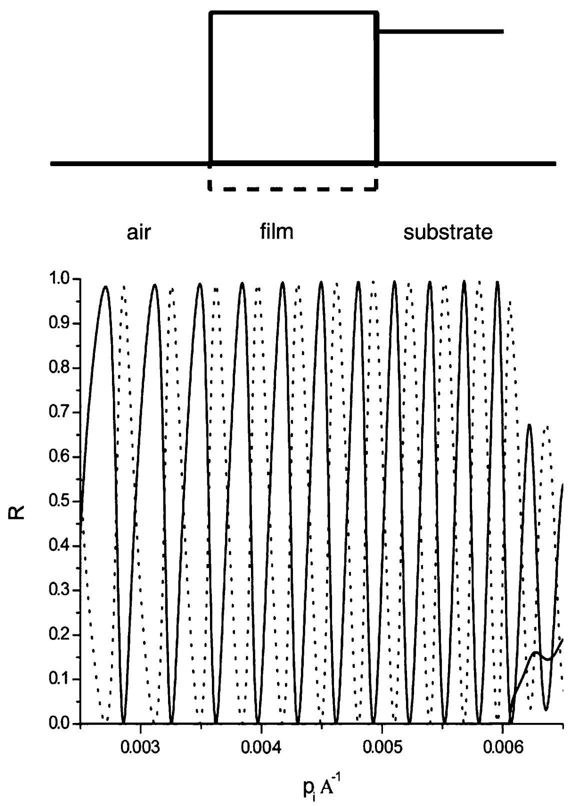

Fig. 3. Oscillations in spin–flip and non-spin–flip intensities reflected from the film magnetized perpendicular to the neutron polarization vector. Calculations produced for the ^{57}Fe film of the thickness $d = 1\mu$ deposited onto the sapphire substrate.

saturating magnetic field of $H = 0.3\,\mathrm{T}$ was applied along an easy axis and then reduced to the remanent field of $H = 3\,\mathrm{G}$. The sample was rotated around the surface normal by 90°. So, the experiment was performed in a guide field of 4 G applied parallel to the surface of the sample and at an angle of 90° to direction of the magnetization of the film. The scheme of the experiment is shown in Fig. 4. Measurements of non-spin–flip, $\mathscr{R}_{++}$ and $\mathscr{R}_{--}$, and spin–flip, $\mathscr{R}_{-+}$, reflectivities where carried out on the PNR reflectometer ADAM [10] at ILL, Grenoble. The experimental data depicted in Fig. 4 are perfectly described by the theoretical curve with the remanent magnetization $M \approx 2\,\mathrm{kG}$ of the iron. Within the reflectivity plateau the ADAM reflectometer allows to well resolve one full oscillation due to pseudo-precession from the sample. Further improvements in the reflectivity techniques [11] would, however, be required to record the effect for either thicker films revealing a number of revolutions, or submonolayers [12] in which the precession angle is small.

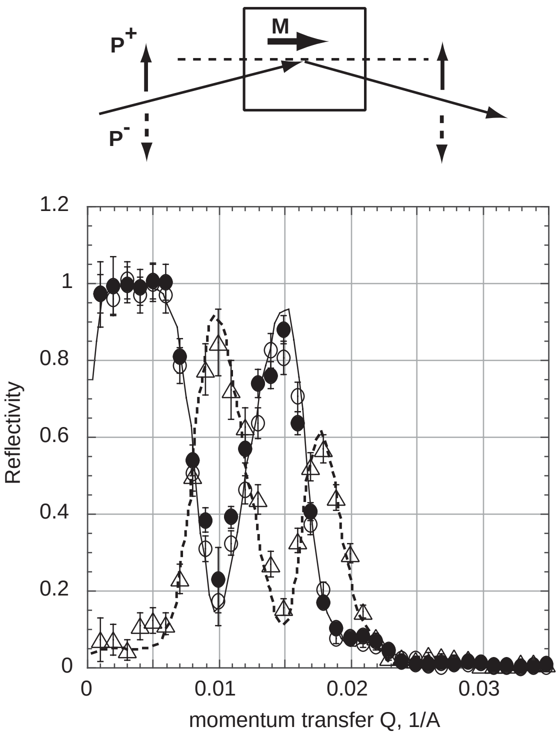

Fig. 4. Oscillations in spin–flip (triangles) and non-spin–flip intensities (circles) reflected from the ^{57}Fe film magnetized perpendicular to the neutron polarization vector.

4. Discussion and summary

The above derivation shows that LP is indeed a result of quantum interference of the spin states split due to the Zeeman effect. This means that LP occurs only within the coherence volume characterizing neutron radiation. On the other hand, LP description has a range of validity restricted by the condition $p_{\mathrm{M}\pm} \ll p_0$, and it fails at wave vectors smaller than a threshold value defined by the magnetic field. It is important to note that the criterium of the LP validity depends on the field configuration. In the kinematics under consideration this criterium is much harder than that followed from just comparison of the neutron energy and the Zeeman splitting. At sufficiently low angles of incidence the threshold of LP applicability can easily be approached and then it receives substantial quantum distortions. Such distortions can readily be observed in experiments on PNR routinely dealing with shallow incidence onto magnetic film. Then the threshold values $p_{\mathrm{M}\pm}$ mentioned above are nothing else than the critical wave vectors of the total reflection for that or the other spin projection onto the field direction.

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