Quantum Afterburner: Improving the Efficiency of an Ideal Heat Engine

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(Received 7 February 2001; revised manuscript received 20 November 2001; published 23 January 2002)

By using a laser and maser in tandem, it is possible to obtain laser action in the hot exhaust gases of a heat engine. Such a "quantum afterburner" involves the internal quantum states of the working molecules as well as the techniques of cavity quantum electrodynamics and is therefore in the domain of quantum thermodynamics. It is shown that Otto cycle engine performance can be improved beyond that of the "ideal" Otto heat engine. Furthermore, the present work demonstrates a new kind of lasing without initial inversion.

DOI: 10.1103/PhysRevLett.88.050602 PACS numbers: 05.10.Gg, 05.40.–a, 42.50.–p

The laws of thermodynamics [1] are very useful in telling us how things work and what things will never work. For example, the ideal heat engine is a paradigm of modern science and technology. We often read in the textbooks the following: "It might be supposed that the ideal cycle analysis is too unrealistic to be useful. In fact, this is not so. Real gas cycles are reasonably close to, although always less efficient than, the ideal cycles."

One goal of the present paper is to reconsider the operating limits of ideal heat engines in light of recent developments in quantum optics such as cavity QED [2], the micromaser [3], and quantum coherence effects such as lasing without inversion (LWI) [4] and cooling via coherent control [5]. In particular, we shall show that by extracting coherent laser radiation from the "exhaust" gas of a heat engine [1], e.g., the Otto cycle idealization of the automobile engine. We here show that it is indeed possible to improve on the efficiency of an ideal Otto cycle engine, operating between two fixed temperature reservoirs, by adding a quantum afterburner which extracts coherent energy from the hot exhaust gases of the heat engine.

Another goal is to demonstrate a new kind of laser operating without initial inversion. In the usual gas dynamic lasers each molecule travels supersonically through the electromagnetic cavity once. In the present scheme, we demonstrate a new kind of laser action in which the lasing gas drifts through the laser cavity due to action of the pistons of Fig. 1c. In such a case the C.M. degrees of freedom are always in thermodynamic equilibrium and thus the atoms bounce off of the cylinder walls passing back and forth through the laser cavity many times as the gas moves from the right to the left-hand side. However, by using both the laser and maser in tandem as indicated in Fig. 1, it is possible to obtain lasing from a thermal population distribution.

The landmark paper by Ramsey [6] on negative temperatures in thermodynamics and statistical mechanics is closely related to the present work. The present study does not involve negative temperature reservoirs. But it is possible to envision a negative temperature as being associated with the $a \to b$ transition once inversion is produced by the maser interaction, and the present work has features in common with that of Ref. [6].

The quantum heat engine concept was introduced by Scovil and Schultz-Dubois [7] and extended in many interesting later works [8]. In the present paper the atomic states are not to be viewed as the engine. We focus on the different problem of improving the efficiency of an Otto engine which contains a laser-maser system integrated into an Otto cycle engine as in Fig. 1.

In what follows we present a physical picture for and thermodynamics of the quantum Otto engine (QOE), including calculation of efficiency and entropy flow, etc. The proposed scheme is simple enough to permit reasonably complete analysis; but, hopefully, realistic enough to be convincing.

In order to present the physics behind the QOE, consider Fig. 1 in which the working fluid passes through the cycle 1234561. As mentioned earlier, we extend the classical Otto engine to include a laser arrangement which can extract coherent laser energy from the internal degrees of freedom. As depicted in Figs. 1a–1f, the QOE operates in a closed cycle in the following steps:

(a) ($1 \to 2$) The hot gas expands isentropically, lifting a weight, and thus doing useful ("good") work $W_g = C_v(T_1 - T_2)$ where $T_2 = T_1\mathcal{R}^{-1}$, C_v is the heat capacity, $\mathcal{R} = (V_1/V_2)^{(\gamma-1)}$, and γ is the ratio of heat capacities at constant pressure to constant volume.

(b) ($2 \to 3$) Heat $Q_{\text{out}} = C_v(T_2 - T_3)$ is extracted at constant volume by a heat exchanger at temperature T_3.

(c) ($3 \to 4$) Maser-laser cavities are added and energy is extracted from the hot internal atomic degrees of freedom by cycling the gas from left to right to left through the laser-maser system held at temperature T_3, with an entropy decrease $\Delta S_{\text{int}} \cong Nk\ln 3$, as discussed later. The function of the maser cavity is to establish a thermal distribution of population between atomic levels $|b\rangle$ and $|c\rangle$ determined by temperature T_3. This will result in a population inversion between $|a\rangle$ and $|b\rangle$ and lasing will take place when the atoms pass into the laser cavity.

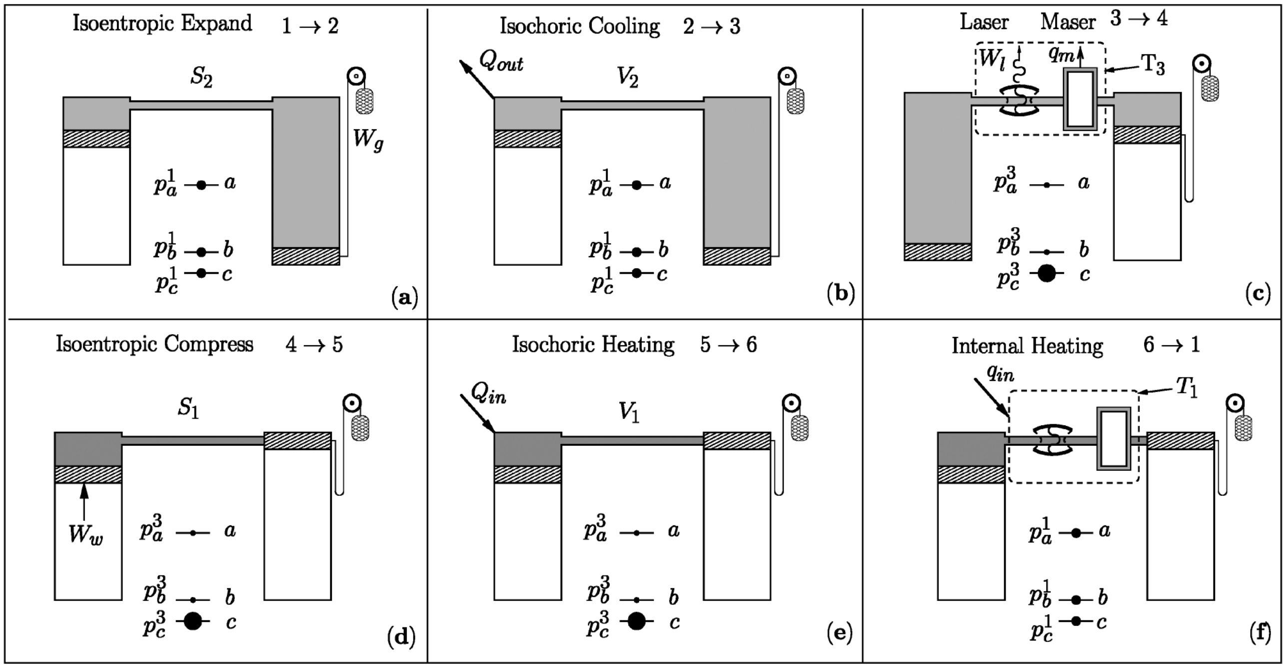

FIG. 1. Illustrated are the steps 1234561 in cyclic operation of the quantum Otto engine. The atomic internal populations are depicted for the three level atom (levels a, b, c) at each stage of operation. A detailed description of operating steps $a \rightarrow f$ is given in the text.

(d) ($4 \rightarrow 5$) The gas is then compressed isentropically to volume $V_4 = V_1$, requiring waste work $W_w = C_v(T_4 - T_3) = (\mathcal{R} - 1)T_3$.

(e) ($5 \rightarrow 6$) The gas is again put in contact with the heat exchanger (at temperature T_1) and the external-translational degrees of freedom are heated isochorically to T_1 by heat energy Q_{in}.

(f) ($6 \rightarrow 1$) Maser-laser cavities are again added and internal states are heated by an amount q_{in} extracted from the hot cavities at temperature T_1, completing the cycle.

As a useful simplifying assumption, we consider the external and internal degrees of freedom to be decoupled. Only when the atoms are passing through the maser-laser system do they change their internal state, that is, the atomic states are chosen to be very long-lived when not in the cavities. But they are strongly coupled to the radiation field in the maser and laser cavities due to the increased density of states of the radiation inside the cavity. Thus, when an atom in the $|b\rangle$ state is passed into the maser cavity it quickly comes into equilibrium with the thermal radiation in the cavity. For example in step, $3 \rightarrow 4$, after passing through the cold maser, $|b\rangle$ state population is governed by the usual Boltzmann distribution determined by temperature T_3. And for small enough T_3 the $|b\rangle$ state is effectively depopulated thus providing a population inversion between states $|a\rangle$ and $|b\rangle$ since the population in state $|a\rangle$ is still determined by T_1. This is the basis for lasing off the thermal energy of the exhaust gases.

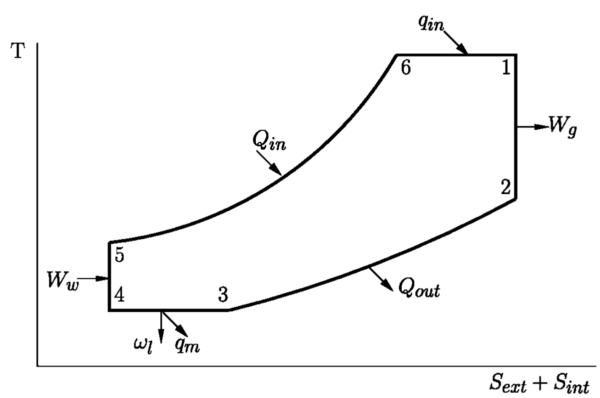

FIG. 2. Shown is a temperature (T) entropy (S) plot for the quantum Otto cycle engine. Note that the entropy is the sum of entropy of the external (kinetic) and internal (quantum) degrees of freedom.

Thus the maser serves as the incoherent ("heat") energy removal mechanism, q_{in}, which enables the coherent (useful) energy, w_l, to be emitted by the laser. To understand the work w_l vs heat q_m aspect of the problem we need only compare the photon statistics for the incoherent thermal field in the maser cavity with the coherent laser field. The population in the states $|b\rangle$ and $|c\rangle$ are not inverted and the maser field density matrix is given by the thermal distribution [9]

$$r_{nn}^{m} = \bar{n}_m^n/(\bar{n}_m + 1)^{n+1}, \tag{1a}$$

where $\bar{n}_m = 1/[\exp(\hbar\nu_m/kT_3) - 1]$, $\hbar\nu_m$ is the energy per quantum of the maser field.

After passing through the maser cavity, there is a population inversion between the $|a\rangle$ and $|b\rangle$ levels. Hence, the density matrix describing the laser field proceeds from

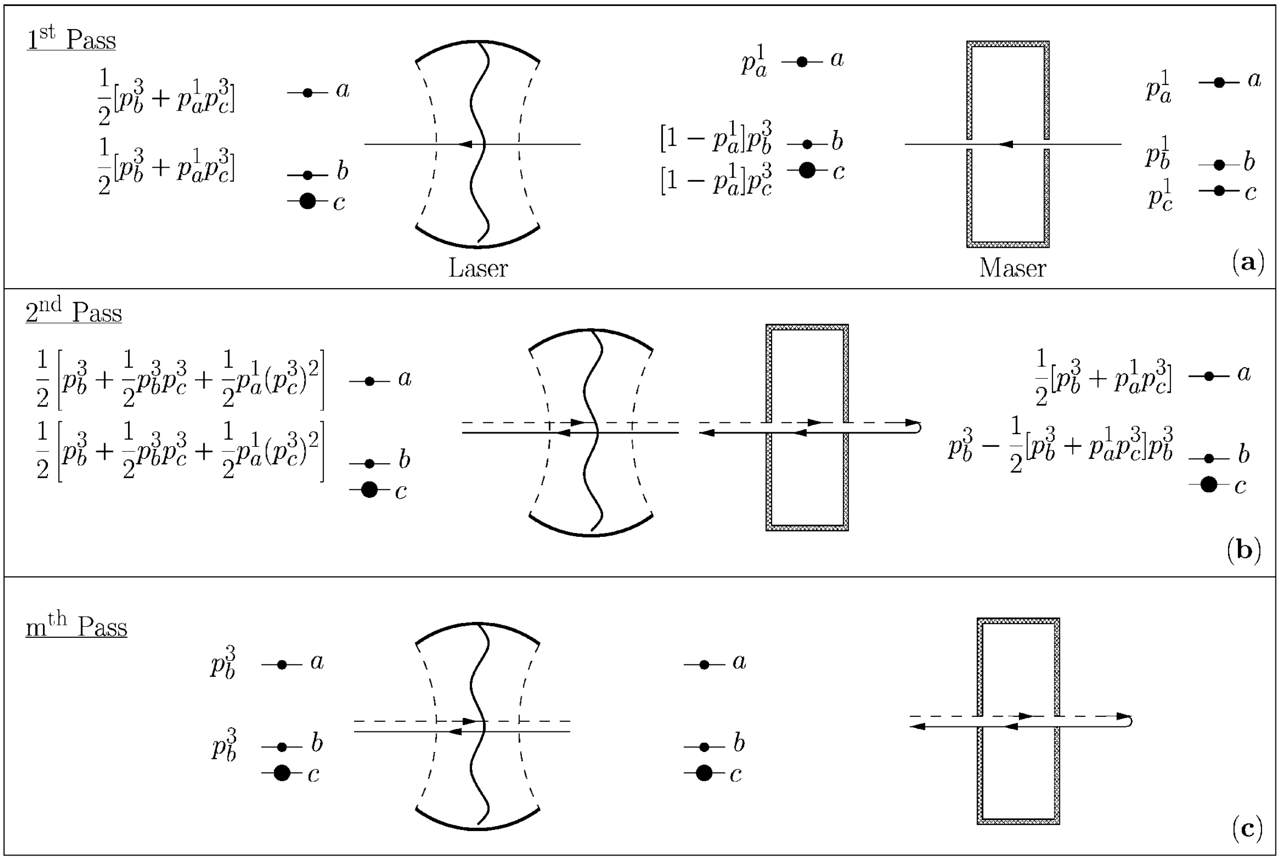

FIG. 3. Figure depicting the evolution of internal atomic populations for the case in which the atom first passes through the maser-laser system; and then (because everything is adiabatic and involves long times and many bounces) bounces back and forth through the cavities many times. As discussed in the text, after a large number of bounces the atom settles down into a configuration wherein most of the population is in state c, with small (equal) amounts in a and b.

an initial thermal state which is largest for small $\bar{n}_l$, to the sharply peaked coherent laser distribution given by [10]

$$r_{n,n}^{(l)} = r_{0,0} \prod_{l=1}^{n} \left[\frac{A(l+1)}{1 + (A/B)(l+1)} - C(l+1) + \frac{\bar{n}_l}{\bar{n}_l + 1} \right], \quad (1b)$$

where $A(C)$ is the linear gain (loss), B is the nonlinear saturation parameter, and $\bar{n}_l$ is the average number of thermal photons in the bare laser cavity at temperature T_3.

Having established that the laser transition $a \to b$ involves extraction of useful work w_l from the atoms, while the maser transition $b \to c$ involves extraction of heat (not work) q_m, we write the efficiency of the QOE as

$$\eta_{q0} = \frac{W_g - W_w + w_l}{Q_{\text{in}} + q_{\text{in}}},$$

and since $q_{\text{in}} = w_l + q_m$ we find

$$\eta_{q0} = \eta_0 + \frac{w_l(1 - \eta_0) - \eta_0 q_m}{Q_{\text{in}} + w_l + q_m}, \quad (2)$$

where the ideal classical Otto engine efficiency is defined as $\eta_0 = (W_g - W_w)/Q_{\text{in}}$. Taking W_g, W_w, and Q_{in} as given in the discussion of QOE operation we have the alternative expressions $\eta_0 = 1 - 1/\mathcal{R} = 1 - T_2/T_1$.

In order to determine whether η_{q0}, Eq. (2), is an improvement over η_0 we now turn to the calculation of w_l and q_m. A rigorous calculation requires a quantum theory of the laser/maser type analysis and this will be given elsewhere. However, it is sufficient for the present purposes to apply microscopic energy balance calculations to obtain good expressions for the important quantities.

After the atom makes one pass through the maser-laser system, the internal density matrix is given by

$$\rho_{\text{one}}(3) = \frac{1}{2}(p_a^1 + p_a^1 p_c^3)(\Lambda_a + \Lambda_b) + \frac{1}{Z_3}(1 - p_a^1)\Lambda_c, \quad (3)$$

where $\Lambda_\alpha = |\alpha\rangle\langle\alpha|, \alpha = a, b,$ and c, and in the notation of Fig. 1, p_α^i is the Boltzmann factor $p_\alpha^i = Z_i^{-1} \exp(-\beta_i \epsilon_\alpha)$ where $\beta_i = 1/kT_i$; T_i is the reservoir temperature T_1 or T_3 and $Z_i = \sum \exp(-\beta_i \epsilon_\alpha)$ [11]. But an atom will bounce many times back and forth through the maser/laser cavities in moving the gas adiabatically from right to left. As is indicated in Fig. 3, after many bounces the atom settles down into the mixed state:

$$\rho_{\text{many}}(3) = p_b^3(\Lambda_a + \Lambda_b) + (1 - 2p_b^3)\Lambda_c, \quad (4)$$

where we have assumed $p_c^3 \gg p_b^3$, and note that $p_a^3 \ll 1$.

We calculate q_m by noting that initially there are $N(p_a^1 + p_b^1)$ atoms in states $|a\rangle$ and $|b\rangle$. After passing through the laser-maser system many times there are only $2Np_b^3$ atoms in $|a\rangle$ and $|b\rangle$. Thus $N(p_a^1 + p_b^1 - 2p_b^3)$ atoms have been added to state $|c\rangle$, and the increase of incoherent maser energy is given by

$$q_m = (\epsilon_b - \epsilon_c)N(p_a^1 + p_b^1 - 2p_b^3). \tag{5}$$

Please note that the incoherent thermal density matrix given by Eq. (1a) assures us that the maser energy q_m is heat energy, i.e., not useful work.

Likewise w_l is obtained by noting that the number of atoms going from a to b with the coherent emission of laser radiation is $N(p_a^1 - p_b^3)$. Energy $\epsilon_a - \epsilon_b$ is given up by each atom, and the total coherent laser energy is

$$w_l = (\epsilon_a - \epsilon_b)N(p_a^1 - p_b^3). \tag{6}$$

The coherent nature of the laser field is embodied in Eq. (1b); see Refs. [9,10].

We now use Eqs. (5) and (6) for q_m and w_l in a form which allows us to determine the sign of the efficiency enhancement factor in Eq. (2), that is, we wish to establish the conditions for which

$$(1 - \eta_0)w_l > \eta_0 q_m. \tag{7a}$$

We use Eqs. (5) and (6) and introduce the notation $\epsilon_{\alpha\beta} = \epsilon_\alpha - \epsilon_\beta$ to write Eq. (7a) as

$$\left(\frac{1}{\eta_0} - 1\right)\left(\frac{\epsilon_{ac}}{\epsilon_{bc}} - 1\right) > 1 + \frac{p_b^1 - p_b^3}{p_a^1 - p_b^3}. \tag{7b}$$

As an example, we may take $\eta_0 = 1/4$, $\epsilon_{ac}/\epsilon_{bc} = 11$ so that the left-hand side (lhs) of Eq. (7b) is 30; furthermore noting that for high enough T_1 that $p_\alpha^1 \simeq 1/3$ and since $p_b^3 \ll p_b^1$, the right-hand side (rhs) of Eq. (7b) equals 2. Hence the rhs (maser) factor is an order of magnitude less than the lhs (laser) factor in (7a) and (7b), which indeed shows that $\eta_{q0} > \eta_0$ as desired. Finally, we note that the von Neumann entropy, $S = -kNTr\rho\ln\rho$, added in the heating of the internal states to temperature T_1:

$$S_{\text{int}}(6 \to 1) = -kN\sum[p_\alpha^1 \ln p_\alpha^1 - 2p_b^3 \ln p_b^3 - p_c \ln p_c], \tag{8}$$

where $p_c = 1 - 2p_b^3$. Hence when T_1 is high enough and T_3 is low enough that $p_c^3 \cong 1$, $p_a^3 \cong p_b^3 \cong 0$ then (8) takes the simple form $S(6 \to 1) \cong kN\ln 3$ as noted earlier.

In conclusion, we have shown that it is possible, in principal, to improve on the efficiency of an ideal Otto cycle engine by extracting laser energy from exhaust gases; this is summarized in Fig. 2. However, as will be presented elsewhere, when a similar "lasing-off-exhaust-heat" scheme is analyzed for a Carnot cycle engine, efficiency is not improved. Suggestions for possible experiments and novel laser schemes will also be presented elsewhere [12].

The author wishes to thank G. Agnolet, H. Bailey, G. Basbas, J. Caton, T. Lalk, S. Lloyd, A. Matsko, F. Narducci, N. Nayak, H. Pilloff, N. Ramsey, Y. Rostovtsev, S. Scully, and M. S. Zubairy for helpful discussions. The support of the Office of Naval Research, the U.S. Air Force, and the Robert A. Welch Foundation is also gratefully acknowledged.

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