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## MAGNETISM AND FERROELECTRICITY

# Domain Walls in Magnetic Multilayers with a Biquadratic Exchange

**V. V. Kostyuchenko**

*Institute of Microelectronics and Informatics, Russian Academy of Sciences, Universitetskaya ul. 21, Yaroslavl, 150007 Russia*

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**Abstract**—The structure of domain walls in magnetic multilayers is investigated taking into account the uniaxial anisotropy and biquadratic exchange between the layers. Analytical solutions are derived for different types of domain wall structures. The majority of the solutions obtained have no analogs in conventional magnetic materials. The thickness and the energy density per unit area are calculated for the domain walls under investigation. The range of parameters that correspond to more energetically favorable structures of domain walls is established. *© 2002 MAIK "Nauka/Interperiodica".*

**1.** At present, the properties of magnetic multilayer structures are under extensive investigation. New materials with magnetic multilayer structures have aroused great interest owing to their unusual physical properties and wide prospects of practical application in memory devices.

Investigations into the phase transitions and the processes of magnetization reversal induced in magnetic superlattices under the action of an external magnetic field are being carried out particularly intensively (see, for example, [1] and references therein). Theoretical studies of phase transitions [2–9] have shown that these materials are characterized by a much greater number of phase transitions as compared to conventional magnetic materials (see, for example, [10]). It was found that the domain walls normal to the plane of layers in magnetic superlattices substantially affect the magnetization reversal in these materials [11–13]; this can lead, in particular, to changes in their resistive characteristics. Furthermore, the domain wall structure in itself can affect the electrical and physical properties of magnetic multilayers. Numerical calculations performed by Labrune and Milat [14] demonstrated that the domain walls in magnetic superlattices possess a number of unusual properties (asymmetry of the domain wall, deviation of the magnetization from the plane of layers, etc.) and could prove a very interesting object of investigation. It was also shown [15] that the symmetric domain walls in magnetic multilayers can be relatively unstable. Morozov and Sigov [16] explained the occurrence of domain walls in multilayers in terms of step-type inhomogeneities at the interface between the magnetic and nonmagnetic layers. However, the existence of domain walls between regions with a noncollinear orientation of magnetization in adjacent layers [11–13] was not interpreted. Moreover, experimental observations of magnetization reversal in magnetic multilayers through the nucleation and growth of domains of the other phase [11–13, 17] also contradict the aforementioned model.

The foregoing shows that the domain walls in magnetic superlattices have not been adequately investigated theoretically. In particular, no consideration is given to the structure of domain walls in the case of noncollinear orientation of the magnetization in adjacent layers, even though such domain walls have been observed experimentally. In the present work, the structure of domain walls in magnetic multilayers is investigated taking into account the biquadratic exchange interaction between the layers for both collinear and noncollinear orientation of the magnetization in adjacent layers. The only case considered is when the magnetization is identical in all the magnetic layers. The situation when the magnetization have different values in the magnetic layers, which is of particular interest, will be considered in a separate publication.

**2.** The domain wall structure is considered in the two-sublattice approximation; i.e., it is assumed that $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetizations in all odd and even layers, respectively ($|\mathbf{M}_1| = |\mathbf{M}_2|$). This approximation holds for a large number of layers [18] and breaks down only in the case of surface spin-flop transition [19]. It should be noted that magnetic multilayer structures are characterized by a greater number of domain wall types as compared to conventional magnets (see, for example, [20]). We assume the thickness of each magnetic layer ($d$) to be smaller than that of the domain wall in the bulk sample ($d \ll \Delta$). Let the $z$ axis be directed along the normal to the plane of the layers. In this case, the dependence of the magnetization on the $z$ coordinate inside each layer can be ignored. Within this approximation, the problem of calculating the dependence of the magnetization $\mathbf{M}(x, y, z, t)$ on three spatial coordinates and time is reduced to the problem of calculating the dependences of two quantities, $\mathbf{M}_1(x, y)$ and

$\mathbf{M}_2(x, y)$, on only two spatial coordinates and time ($\mathbf{M}_i$ is the magnetization in the $i$th magnetic layer).

For a domain wall in a two-layer magnetic structure, we write the variational principle in the following form:

$$\delta \int F_S dS = 0. \quad (1)$$

It is appropriate to express the energy density $F_S$ in terms of angular variables $\theta_i$ and $\varphi_i$, which determine the orientation of the magnetization in the $i$th magnetic layer. The polar angles $\theta_i$ are reckoned from the $z$ axis, and the azimuthal angles $\varphi_i$ are measured from the $x$ axis in the $xy$ plane.

The energy density $F_S$ in angular variables $\theta_i$ and $\varphi_i$ takes the form

$$F_S = \sum_{i=1}^{2} \left\{ \frac{1}{2} K \sin^2\theta_i \sin^2\varphi_i + 2\pi M^2 \cos^2\theta_i + \frac{1}{2} A[(\nabla\theta_i)^2 + \sin^2\theta_i (\nabla\varphi_i)^2] \right\}$$
$$+ \frac{1}{2} J_1 (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2))$$
$$+ \frac{1}{2} J_2 (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2))^2, \quad (2)$$

where $\gamma$ is the gyromagnetic ratio, $M$ is the saturation magnetization in the ferromagnetic layer, $H$ is the external magnetic field directed along the $x$ axis, $K$ is the uniaxial anisotropy constant, $A$ is the inhomogeneous exchange constant, and $J_1$ and $J_2$ are the constants of the Heisenberg and biquadratic exchange between the magnetic layers, respectively. Relationship (2) is obtained from the Lagrangian density for a single-sublattice ferromagnet with inclusion of the exchange interaction between the magnetic layers.

Since the demagnetization energy inhibits the deviation of the magnetization from the plane of the layers, we can assume that $\theta_i = \text{const} = \pi/2$. In this case, it is convenient to introduce the variables $\varphi$ and $\psi$:

$$\begin{cases} \varphi_1 = \varphi + \psi \\ \varphi_2 = -\varphi + \psi, \end{cases} \quad \begin{cases} \varphi_1 - \varphi_2 = 2\varphi \\ \varphi_1 + \varphi_2 = 2\psi. \end{cases} \quad (3)$$

Consequently, the functional $F_S$ can be rewritten as

$$F_S = K[\sin^2\varphi + \sin^2\psi \cos 2\varphi] + A[(\nabla\varphi)^2 + (\nabla\psi)^2] + \frac{1}{2} J_1 \cos 2\varphi + \frac{1}{2} J_2 \cos^2 2\varphi \quad (4)$$

and equations used to describe the domain wall structure take the form

$$K \sin 2\psi \cos 2\varphi = 2A\nabla^2\psi, \quad (5a)$$

$$K \sin 2\varphi \cos 2\psi - J_1 \sin 2\varphi - J_2 \sin 4\varphi = 2A\nabla^2\varphi. \quad (5b)$$

For definiteness, we assume that the domain wall plane is perpendicular to the $z$ axis. In the case of a planar domain wall, the system of equations (5) has the first integral

$$K[\sin^2\varphi + \sin^2\psi \cos 2\varphi] + \frac{1}{2} J_1 \cos 2\varphi + \frac{1}{2} J_2 \cos 2\varphi + \text{const} = A[(\varphi')^2 + (\psi')^2]. \quad (6)$$

This allows us to find analytical solutions to this system of equations.

In order to determine unambiguously the domain wall structure, it is necessary to complement the system of equations (5) with the boundary conditions. The boundary conditions can be determined from the condition of stability of the homogeneous state. The minimum condition of the functional for the homogeneous state can be met with the following four phases depending on the magnitudes of the Heisenberg and biquadratic exchange interactions between the layers (see, for example, [7]): the ferromagnetic and antiferromagnetic phases, the angular phase with the magnetization vectors in adjacent layers that are symmetrically oriented with respect to the easy axis, and the angular phase with the magnetization vectors in adjacent layers that are symmetrically oriented with respect to the hard axis.

**3.** At $J_1 < 0$ and $K > J_1 > 2J_2$, the minimum energy corresponds to the ferromagnetic phase. In this case, two types of domain walls become possible, namely, types 1$a$ and 1$b$ (Fig. 1). The rotation of magnetization in adjacent layers occurs in the same direction for type 1$a$ and in opposite directions for type 1$b$. The boundary conditions for the type 1$a$ domain walls have the form

$$\varphi = \text{const} = 0,$$
$$\psi(-\infty) = 0, \quad \psi(+\infty) = \pi, \quad \psi'(\pm\infty) = 0. \quad (7)$$

The solution to the system of equations (4) with the boundary conditions (7) takes the form

$$\psi = 2\arctan\exp(x/\Delta_0), \quad (8)$$

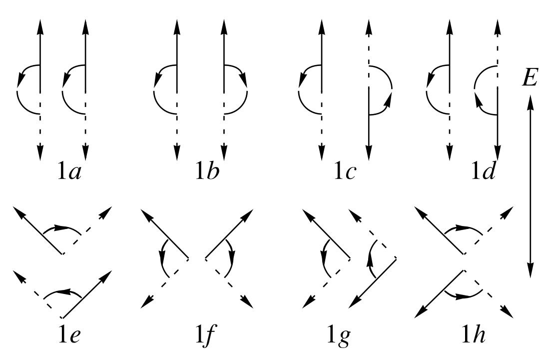


**Fig. 1.** The sense of rotation of the magnetization vector in adjacent layers for different types of domain walls. $E$ is the easy axis.

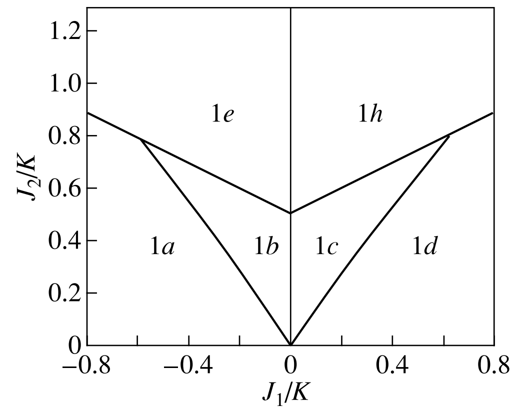


**Fig. 2.** The regions of parameters corresponding to the minimum energy for different types of domain walls shown in Fig. 1.

where $\Delta_0 = \sqrt{A/K}$. For domain walls 1*b*, the boundary conditions are represented as

$$\psi = \text{const} = 0,$$
$$\varphi(-\infty) = 0, \quad \varphi(+\infty) = \pi, \quad \varphi'(\pm\infty) = 0. \tag{9}$$

These conditions are satisfied with the solution

$$\varphi = \arccos\left[\frac{-\tanh(x\sqrt{1-q_f}/\Delta_f)}{\sqrt{1-q_f/\cosh^2(x\sqrt{1-q_f}/\Delta_f)}}\right], \tag{10}$$

where $\Delta_f = \sqrt{A/(K-J_1)}$ and $q_f = 2J_2/(K-J_1)$.

**4.** The calculated wall energies per unit area for different types of domain walls are presented in the table. If the magnitude of the exchange interaction between the layers is large [$|J_1| > (16/\pi^2 - 1)K$], the energy of the domain wall 1*a* is higher than that of the domain wall 1*b*. For $|J_1| < (16/\pi^2 - 1)K$, the lower energy can be observed for either the type 1*a* or the type 1*b* domain wall, depending on the magnitude of the biquadratic exchange interaction. The regions of parameters corresponding to different types of domain walls are shown in Fig. 2.

Calculated energies per unit area for different types of domain walls shown in Fig. 1

| | |
|---|---|
| 1*a* | $4\sqrt{AK}$ |
| 1*b* | $2\sqrt{A(K-J_1)}\left(\sqrt{1-q_f} + \frac{1}{\sqrt{q_f}}\arcsin\sqrt{q_f}\right)$ |
| 1*c* | $4\sqrt{AK}$ |
| 1*d* | $2\sqrt{A(K-J_1)}\left(\sqrt{1-q_a} + \frac{1}{\sqrt{q_a}}\arcsin\sqrt{q_a}\right)$ |
| 1*e* | $2\sqrt{2AJ_2}(\sin 2\varphi_I + 2\varphi_I \cos 2\varphi_I)$ |
| 1*f* | $2\sqrt{2AJ_2}(\sin 2\varphi_I + (\pi - 2\varphi_I)\cos 2\varphi_I)$ |
| 1*g* | $2\sqrt{2AJ_2}((\pi - 2\varphi_{II})\cos 2\varphi_{II} - \sin 2\varphi_{II})$ |
| 1*h* | $2\sqrt{2AJ_2}(2\varphi_{II}\cos 2\varphi_{II} - \sin 2\varphi_{II})$ |

**5.** In the case when the antiferromagnetic phase is more energetically favorable ($K + J_1 > 2J_2$, $J_1 > 0$), there can also exist two types of domain walls that differ in the sense of rotation of the magnetization in adjacent layers, namely, types 1*c* and 1*d* (Figs. 1*c*, 1*d*). The boundary conditions for domain walls 1*c* are given by

$$\varphi = \text{const} = \pi/2,$$
$$\psi(-\infty) = -\pi/2, \quad \psi(+\infty) = \pi/2, \quad \psi'(\pm\infty) = 0. \tag{11}$$

The solution to the system of equations (5) with the boundary equations (11) has the form

$$\psi = 2\arctan\exp(x/\Delta_0). \tag{12}$$

For domain walls 1*d*, the boundary equations are represented as

$$\psi = \text{const} = \pi/2,$$
$$\varphi(-\infty) = -\pi/2, \quad \varphi(+\infty) = \pi/2, \quad \varphi'(\pm\infty) = 0. \tag{13}$$

These conditions are satisfied with the following solution to the system of equations (5):

$$\varphi = \arcsin\left[\frac{\tanh(x\sqrt{1-q_a}/\Delta_a)}{\sqrt{1-q_a/\cosh^2(x\sqrt{1-q_a}/\Delta_a)}}\right], \tag{14}$$

where $\Delta_a = \sqrt{A/(K+J_1)}$ and $q_a = 2J_2/(K + J_1)$.

As in the case of the ferromagnetic phase when the magnitude of the Heisenberg exchange interaction between the layers is small [$|J_1| < (16/\pi^2 - 1)K$], the energy of the domain wall 1*d* with opposite directions of the magnetization vectors in adjacent layers can be less than that of the domain wall 1*c*. The region of parameters $J_1$ and $J_2$, which corresponds to this situation, is displayed in Fig. 2.

**6.** For $J_1 < 0$ and $K < J_1 + 2J_2$, the minimum energy corresponds to the angular phase with symmetric orientation of the magnetization vectors in adjacent layers with respect to the easy axis. The possible types of domain walls for this case are represented in Figs. 1*e* and 1*f*. The boundary conditions for domain walls 1*e* have the following form:

$$\psi = \text{const} = 0,$$
$$\varphi(-\infty) = -\varphi_I, \quad \varphi(+\infty) = \varphi_I, \quad \varphi'(\pm\infty) = 0. \tag{15}$$

The solution to the system of equations (5), which satisfies these boundary conditions, is given by the formula

$$\varphi = \arctan[\tan\varphi_I \tanh(x\sin 2\varphi_I/\Delta_1)], \tag{16}$$

where $\Delta_1 = \sqrt{2A/J_2}$ and $\varphi_I = \frac{1}{2}\arccos\frac{K-J_1}{2J_2}$.

For domain walls 1*f*, the boundary conditions are given by

$$\varphi(-\infty) = \varphi_I, \quad \varphi(+\infty) = \pi - \varphi_I, \quad \varphi'(\pm\infty) = 0. \tag{17}$$

In this case, we obtain the following solutions to the system of equations (5):

$$\begin{gathered} \psi = \text{const} = 0, \\ \varphi = \operatorname{arccot}[\cot\varphi_{\rm I}\tanh(x\sin 2\varphi_{\rm I}/\Delta_1)]. \end{gathered} \tag{18}$$

The energy of the domain wall 1*e* is less than that of domain wall 1*f*, because the energy of the biquadratic exchange interaction is identical for both types of domain walls, whereas the energy of anisotropy and the energy of the Heisenberg exchange interaction for domain wall 1*e* are less than those for the domain wall 1*f*.

**7.** For $J_1 > 0$ and $K + J_1 < 2J_2$, the minimum energy corresponds to the angular phase with symmetric orientation of the magnetization in adjacent layers with respect to the hard axis. The possible types of domain wall structures for this case are shown in Figs. 1*g* and 1*h*. The boundary conditions for domain walls 1*g* have the form

$$\begin{gathered} \psi = \text{const} = \pi/2, \quad \varphi(-\infty) = \varphi_{\rm II} - \pi/2, \\ \varphi(+\infty) = \pi/2 - \varphi_{\rm II}, \quad \varphi'(\pm\infty) = 0, \end{gathered} \tag{19}$$

where $\varphi_{\rm II} = \frac{1}{2}\arccos\frac{K + J_1}{2J_2}$. The solution to the system of equations (5), which satisfies the boundary conditions (19), is represented by the formula

$$\varphi = \arctan[\cot\varphi_{\rm II}\tanh(x\sin 2\varphi_{\rm II}/\Delta_2)]. \tag{20}$$

Similarly, the boundary conditions for domain walls 1*h* have the form

$$\begin{gathered} \psi = \text{const} = \pi/2, \quad \varphi(-\infty) = \pi/2 - \varphi_{\rm II}, \\ \varphi(+\infty) = \pi/2 + \varphi_{\rm II}, \quad \varphi'(\pm\infty) = 0. \end{gathered} \tag{21}$$

The solution satisfying these conditions is given by

$$\varphi = \operatorname{arccot}[\tan\varphi_{\rm II}\tanh(x\sin 2\varphi_{\rm II}/\Delta_1)]. \tag{22}$$

As in the preceding case, the energy of the domain wall 1*h* is always less than that of the domain wall 1*g*, because the energy of the Heisenberg exchange interaction and the energy of anisotropy for the former structure are also less than those for the latter structure.

**8.** Thus, the structure of domain walls in magnetic superstructures is investigated. Eight exact solutions are obtained for different types of domain walls. The sense of rotation of the magnetization vectors in adjacent layers is represented in Fig. 1. The domain wall structures 1*b* and 1*d*–1*h* have no analogs in conventional magnetic materials. Domain walls of types 1*f* and 1*g* are universally characterized by a higher energy compared to that of domain walls of types 1*e* and 1*h*, respectively. However, it should be noted that the inclusion of the magnetostatic energy can change this ratio.


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## REFERENCES


1. I. K. Schuller, S. Kim, and C. Leiton, J. Magn. Magn. Mater. **200** (1–3), 571 (1999).
2. A. K. Zvezdin and S. N. Utochkin, Pis'ma Zh. Éksp. Teor. Fiz. **57** (7), 424 (1993) [JETP Lett. **57**, 439 (1993)].
3. S. N. Utochkin and A. K. Zvezdin, J. Magn. Magn. Mater. **140–144**, 787 (1995).
4. A. K. Zvezdin and S. N. Utochkin, Pis'ma Zh. Éksp. Teor. Fiz. **57** (7), 418 (1993) [JETP Lett. **57**, 433 (1993)].
5. R. W. Wang and D. L. Mills, Phys. Rev. B **50** (6), 3931 (1994).
6. N. S. Almeida and D. L. Mills, Phys. Rev. B **52** (18), 13504 (1995).
7. A. K. Zvezdin and V. V. Kostyuchenko, Fiz. Tverd. Tela (St. Petersburg) **41** (3), 461 (1999) [Phys. Solid State **41**, 413 (1999)].
8. V. V. Kostyuchenko and A. K. Zvezdin, J. Magn. Magn. Mater. **176** (1), 155 (1997).
9. V. V. Kostyuchenko and A. K. Zvezdin, Phys. Rev. B **57** (10), 5951 (1998).
10. A. K. Zvezdin, in *Handbook of Magnetic Materials*, Ed. by K. H. J. Buschow (Elsevier, Amsterdam, 1995), Vol. 9, p. 405.
11. V. I. Nikitenko, L. M. Dedukh, V. S. Gornakov, *et al.*, IEEE Trans. Magn. **33** (5), 3661 (1997).
12. V. I. Nikitenko, V. S. Gornakov, L. M. Dedukh, *et al.*, Phys. Rev. B **57** (14), R8111 (1998).
13. V. I. Nikitenko, V. S. Gornakov, L. M. Dedukh, *et al.*, J. Appl. Phys. **83** (11), 6828 (1998).
14. M. Labrune and J. Milat, J. Magn. Magn. Mater. **151** (1–2), 231 (1995).
15. A. K. Zvezdin and V. V. Kostyuchenko, Zh. Éksp. Teor. Fiz. **116** [4(10)], 1365 (1999) [JETP **89**, 734 (1999)].
16. A. I. Morozov and A. S. Sigov, Pis'ma Zh. Éksp. Teor. Fiz. **61** (11), 893 (1995) [JETP Lett. **61**, 911 (1995)].
17. S. G. E. te Velthuis, J. S. Jiang, and G. P. Felcher, Appl. Phys. Lett. **77** (14), 2222 (2000).
18. A. K. Zvezdin and V. V. Kostyuchenko, Fiz. Tverd. Tela (St. Petersburg) **39** (1), 178 (1997) [Phys. Solid State **39**, 155 (1997)].
19. R. W. Wang, D. L. Mills, E. E. Fullerton, *et al.*, Phys. Rev. Lett. **72** (6), 920 (1994).
20. K. P. Belov, A. K. Zvezdin, A. M. Kadomtseva, and R. Z. Levitin, *Orientational Transitions in Rare-Earth Magnets* (Nauka, Moscow, 1979).


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