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Magnetism in Ferromagnetic Spin-1/2 Ising Multilayers

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In this work, we study the magnetization profiles of ferromagnetic multilayers consisting of two materials in a sandwich structure (ABA), with ferromagnetic coupling between the films, using an effective field theory with a probability distribution technique that accounts for the self-correlation function. Different exchange interaction terms at the interface are investigated. We present also results on the effect of both the exchange interaction constant in material B and the surface exchange interaction constant on the whole system.

1. Introduction

Magnetic multilayers have gained increasing for fundament research and technological applications (see for a detailed review [1]). Their magnetic behavior is an important area of research in solid state physics as well as in certain industrial fields such as micro-electronics and magneto-optics recording [2 to 4].

Multilayered structures composed of two or more different ferromagnetic layers have already been artificially fabricated. The recent progress in preparation and characterization techniques enables one to produce ultra-thin layered structure, in which the sublayers are controlled up to atomic scale. The critical properties of such systems have been studied both experimentally [2 to 19] and theoretically [20 to 40].

Much work has been done to investigate multilayers consisting of different ferromagnetic materials coupled ferromagnetically or antiferromagnetically. Fishman et al. [23] investigated a periodic multilayer consisting of two ferromagnetic materials using a general Ginsburg-Landau theory for inhomogeneous systems and the appropriate equation of motion. They obtained a phase diagram (T_c versus thickness of the layers) and the temperature dependence of the magnetization and the magnon dispersion relation for the composite multilayers EuS/EuO. Camley et al. [24 to 26] with the help of a self-consistent mean field calculation found many phases of a multilayer consisting of ferromagnetic materials coupled antiferromagnetically. Later, LePage and Camley [27, 28] investigated the spin wave spectrum of these systems. All calculations were done with the special parameters to describe the Fe/Gd system, where the ferromagnetic layers Fe and Gd are coupled antiferromagnetically. Recent experiments on Fe/Gd multilayers show that the predicted phases really exist [5, 6].

For more complicated superlattices with arbitrary number of different layers in an elementary unit, Barnás [29, 30] has derived some general dispersion equations for the bulk and surface magnetic polaritons. These equations were then applied to magnetostatic modes and to retarded wave propagation in the Voigt geometry [31].

From both experimental and theoretical points of view, the spin-1/2 Ising magnetic superlattice is very important. Aguilera-Granja et al. [36] modelled some experiment results by the ferromagnetic thin film within the framework of the spin-1/2 Ising Hamiltonian in the mean field approximation. As a result, analytical expression of the thickness dependence of the critical temperature was obtained and the model was applied to estimate magnetic interaction in Fe and Gd ultra-thin films. The experimental results in the case of Fe on Au substrates [10] were very well reproduced. Hai et al. [37] and Bouziane et al. [38] used the effective field theory to study the critical properties in a superlattice consisting of two ferromagnetic materials with different bulk transition temperatures. Ez-Zahraouy [39] by means of a spin-1/2 Ising model in the effective field theory with a finite cluster approximation technique, studied the magnetic properties of a three-layer system of two spin-1/2 with different bulk properties and amorphous interface layers. His results are in agreement with those obtained by Liu et al. [11] on the (Fe−Si)/Pd multilayered film. On the other hand, there exists a class of magnetic substances that have been extensively studied (mainly: Co/Ru, Co/Cr, Fe/Cr [7], Fe/Cr [8] Fe/Gd [5, 6, 32, 33], Fe/Cr/Fe [25], Fe/Ag [20], etc.) and for many of these materials theoretical models were proposed.

Recently Bouziane et al. [42] have used the effective field theory with a probability technique to study the critical behavior of a sandwich multilayer in the pure case of a spin-1/2 Ising multilayer [42], but there still has been no discussion about the magnetization behavior, therefore we shall discuss this subject hereafter.

In this paper we study the qualitative ferromagnetic behavior of a system consisting of two materials A and B in a sandwich structure ABA. We use the effective field theory with a probability distribution technique [40, 41]. This method takes into consideration the fluctuation of the effective field and correctly accounts for all the single kinematic relations.

The exchange coupling interaction in material B is lower than in material A. In particular, we study the influence of the interface ferromagnetic coupling exchange constant between the films, and of the coupling exchange constant of the material B, on the magnetic behavior of the whole system. We have also investigated the influence of the surface exchange coupling constant on the magnetic behavior of the multilayer. We discuss the temperature dependence of the magnetization, focusing on the zero-field behavior on the system.

In Section 2 we outline the effective field theory and derive the equations that determine the transition temperature and give the equation of the magnetization. Details of this method are given in [38, 42]. In Section 3 we present and discuss the result of our calculations.

2. Theory

We consider a multilayer consisting of three ferromagnetics films with a simple cubic structure where each material of four atomic layers in the plane (x, y). The first and the third films are denoted as A and the second film is denoted as B. The situation is shown in Fig. 1. The coupling strength between nearest-neighbor spins in A(B) is denoted by $J_a(J_b)$. It takes the value J_s if both spins are nearest neighbors within the surface layers (top and bottom surfaces), the value J_1 if it is between a spin on the surface and its nearest neighbor in the next layer, while J_{ab} stands for the exchange

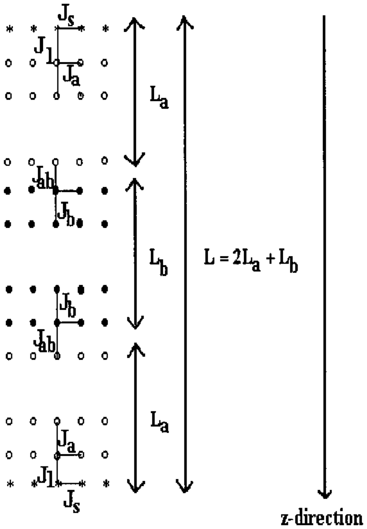

Fig. 1. Two-dimensional cross section of the multilayer composed of ferromagnetic materials A and B, where $L = 2L_a + L_b$ is the thickness of the multilayer

coupling between the nearest-neighbor spins across the interface. The corresponding number of atomic layers in A(B) is $L_a = 4$ ($L_b = 4$) and the thickness of the multilayer is $L = 2L_a + L_b$. The Hamiltonian of the system is given by

$$H = -\sum_{\langle i,j \rangle} J_{i,j}\sigma_i^z\sigma_j^z - \sum_i \Omega_i\sigma_i^x , \qquad (1)$$

where the first sum runs over all nearest-neighbor pairs and the second sum is taken over all the spins, σ_i^z and σ_i^x denote the z and x components of a quantum spin σ_i of magnitude $\sigma = 1/2$ at site i, $J_{i,j}$ stands for one of the five coupling constants depending on where the spin pair is located, and Ω_i represents the transverse field within material A(B) which will be denoted by $\Omega_a(\Omega_b)$, throughout the calculations Ω_i will be set to zero.

The statistical properties of the system are studied using an effective field theory whose starting point is the generalized, but approximate Callen [43] relation derived by Sá Barreto et al. [44] for the transverse Ising model. In order to derive the effective field equations for the magnetizations, we consider a cluster comprising just a single selected spin, labeled i, and the neighboring spins with which it directly interacts. To this end, the Hamiltonian is split into two parts, $H = H_i + H'$, where H_i is that part of the Hamiltonian containing the spin i, namely

$$H_i = -\left[\left(\sum_{j=1}^{Z} J_{i,j}\sigma_j^z\right)\sigma_i^z + \Omega_i\sigma_i^x\right] , \qquad (2)$$

where $Z = N + 2N_0$ is the number of nearest neighbors of the lattice site i (for details see [41]).

The starting point of the effective field theory is a set of formal identities of the type

$$\langle\langle\sigma_i^\alpha\rangle_c\rangle = \left\langle\frac{\mathrm{Tr}_i\,[\sigma_i^\alpha \exp(-\beta H_i)]}{\mathrm{Tr}_i\,[\exp(-\beta H_i)]}\right\rangle , \qquad (3)$$

where $\alpha = z, x$, $\langle\sigma_i^\alpha\rangle_c$ denotes the mean value of σ_i^α for a given configuration c of all other spins, i.e. when all other spins $\sigma_j (j \neq i)$ have fixed values, $\langle\ldots\rangle$ denotes the average over all spin configurations σ_j, Tr_i means the trace performed over σ_i only, $\beta = 1/k_B T$ with k_B the Boltzmann constant (we take $k_B = 1$ for simplicity) and T the absolute temperature.

Because of the approximation made by Sá Barreto and Fittipaldi [44, 45] to obtain the term between brackets on the right hand side, the Eqs. (3) are no longer exact for an Ising system in a transverse field but they have nevertheless been accepted as a reasonable starting point in many studies of that system.

To evaluate $\langle\sigma_i^\alpha\rangle_c$, one has to expand the inner traces in Eqs. (3) over the states of the spin i. This yields

$$m_i^\alpha = \langle\langle\sigma_i^\alpha\rangle_c\rangle = \left\langle f_\alpha\left(\sum_{J=1}^{Z} J_{i,j}\sigma_j^z,\, \Omega_i\right)\right\rangle, \tag{4}$$

where

$$f_z\left(\sum_{j=1}^{Z} J_{i,j}\sigma_j^z,\, \Omega_i\right) = \frac{1}{2}\,\frac{\sum_{j=1}^{Z} J_{i,j}\sigma_j^z}{\left[\left(\sum_{j=1}^{Z} J_{i,j}\sigma_j^z\right)^2 + \Omega_i^2\right]^{1/2}}\tanh\left(\frac{1}{2}\,\beta\left[\left(\sum_{j=1}^{Z} J_{i,j}\sigma_j^z\right)^2 + \Omega_i^2\right]^{1/2}\right) \tag{5}$$

and

$$f_x\left(\sum_{j=1}^{Z} J_{i,j}\sigma_j^z,\, \Omega_i\right) = f_z\left(\Omega_i,\, \sum_{j=1}^{Z} J_{i,j}\sigma_j^z\right). \tag{6}$$

The sum in Eqs. (4 to 6) is over the Z nearest neighbors of the site i. For the case of a simple cubic lattice which we will consider here, one has $N = 4$, $N_0 = 1$ and $Z = N + 2N_0$, N and N_0 denote respectively the numbers of nearest neighbors in the plane and between adjacent planes. In a mean field approximation one would simply replace these spin operators σ_j^z by their thermal values (the longitudinal magnetization).

However, it is at this point that a substantial improvement to the theory is made by noting that the spin operators σ_j^z have a finite set of base states, so that average over the function f_α can be expressed as an average over a finite polynomial of the spin operators σ_j^z belonging to the neighboring spins. This procedure can be performed by the combinatorial method and correctly accounts for the single site kinematics relations, but the right-hand side of Eq. (4) will contain multiple spin correlation functions.

It is worth noting that the theory used here has a short-coming that is common to all mean-field-like methods, namely that it is necessary to identify the order parameter beforehand and it is essential for an approximation to work that the spins have a number of states each, in order to provide a finite algebra.

For simplicity, we assume that the transverse field depends only on the layer index, which we shall denote by n. Because of the translational symmetry parallel to the (001) plane, the magnetization depends only on n. To perform thermal averaging on the right-hand side of Eq. (4), we follow the general approach described in [40, 41].

First of all, in the spirit of the effective field theory, multi-spin correlation functions are approximated by products of single spin averages. We then take advantage of the integral representation of Dirac's delta distribution, in order to write Eq. (4) in the form

$$m_n^\alpha = \int \mathrm{d}\omega\, f_\alpha(\omega,\, \Omega_n)\,\frac{1}{2\pi}\int \mathrm{d}t\,\exp\,(i\omega t)\prod_j \langle\exp\,(-itJ_{i,j}\sigma_j^z)\rangle\,. \tag{7}$$

We now introduce the probability distribution of the spin variables (for details see Tucker et al. [40] and Saber [41])

$$P(\sigma_n^z) = \tfrac{1}{2}\,[(1 - 2m_n^z)\,\delta(\sigma_n^z + \tfrac{1}{2}) + (1 + 2m_n^z)\,\delta(\sigma_n^z - \tfrac{1}{2})]\,. \tag{8}$$

Using this expression and Eq. (7) we obtain the following set of equations for the longitudinal and transverse magnetizations of layer n:

$$m_1^\alpha = 2^{-N-N_0} \sum_{K=0}^{N} \sum_{K_1=0}^{N_0} \{C_K^N C_{K_1}^{N_0} (1-2m_1^z)^K \, (1+2m_1^z)^{N-K} \times (1-2m_2^z)^{K_1} \, (1+2m_2^z)^{N-K_1} f_\alpha(\tfrac{1}{2}\, [J_{1,1}(N-2K)+J_{1,2}(N_0-2K_1)],\, \Omega_1)\}\,, \tag{9}$$

for $n = 2, \ldots, L-1$

$$m_n^\alpha = 2^{-N-2N_0} \sum_{K=0}^{N} \sum_{K_1=0}^{N_0} \sum_{K_2=0}^{N_0} \{C_K^N C_{K_1}^{N_0} C_{K_2}^{N_0} (1-2m_n^z)^K \, (1+2_n^z)^{N-K} \times (1-2m_{n-1}^z)^{K_1} \, (1+2m_{n-1}^z)^{N-K_1} \, (1-2m_{n+1}^z)^{K_2} \, (1+2m_{n+1}^z)^{N-K_2} \times f_\alpha(\tfrac{1}{2}\, [J_{n,n}(N-2K_1)+J_{n,n-1}(N_0-2K_1)+J_{n,n+1}(N_0-2K_2)],\, \Omega_n)\} \tag{10}$$

and (for $n = L$)

$$m_L^\alpha = 2^{-N-N_0} \sum_{K=0}^{N} \sum_{K_1=0}^{N_0} \{C_K^N C_{K_1}^{N_0} (1-2m_L^z)^K \, (1+2m_L^z)^{N-K} \times (1-2m_{L-1}^z)^{K_1} \, (1+2m_{L-1}^z)^{N-K_1} f_\alpha(\tfrac{1}{2}\, [J_{L,L}(N-2K)+J_{L,L-1}(N_0-2K_1)],\; \Omega_L)\}\,, \tag{11}$$

where $\alpha = z, x$.

We recall that in these equations, N and N_0 denote respectively the numbers of nearest neighbors in the plane and between adjacent planes, and C_k^l are the binomial coefficients, $C_k^l = l!/(k!(l-k)!)$. For the case of a simple cubic lattice which we will consider here, one has $N = 4$, $N_0 = 1$ and $Z = N + 2N_0$.

We have thus obtained the self-consistent equations (9 to 11) for the longitudinal and transverse magnetizations of a layer that can be solved directly by numerical iteration. However, if we are interested in the calculations of the longitudinal order near the critical temperature, the usual argument that the longitudinal layer magnetization m_n^z tends to zero as the temperature approaches its critical value, allows us to consider only linear terms in m_n^z, because higher order terms tend to zero faster than m_n^z on approaching the critical temperature. In what follows, we take $J_a = J = 1$ as the unit of energy and in our numerical calculations length is measured in units of the lattice constant.

3. Results and Discussion

For an Ising multilayer consisting of three materials with a simple cubic structure the longitudinal layer magnetization is function of temperature and of exchange interaction in each material A and B. It also depends on interface and surface exchange interactions and on material thicknesses.

Solving equations (9 to 11) numerically, we obtained the value of the layer magnetization and the average magnetization M of the multilayer: $M = \frac{1}{12} \sum_{i=1}^{12} m_i$ (here we set

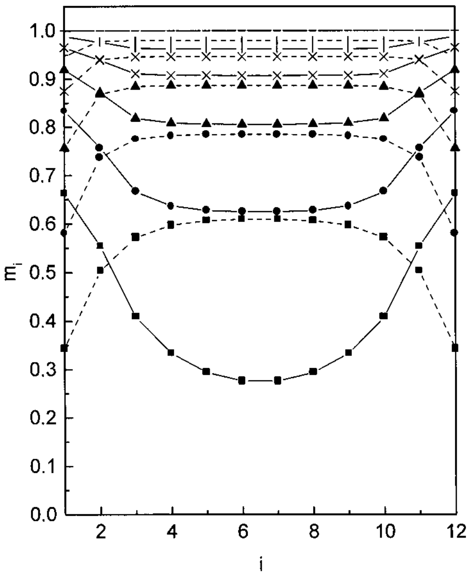

Fig. 2. The magnetization profiles of a simple film at different temperature $T/T_c = 0.9$ (■), 0.8 (●), 0.7 (▲), 0.6 (×), 0.5 (|), 0.1 (–). Solid and dashed lines corresponding to the cases $R_s = R_1 = 1.50$ and $R_s = R_1 = 1.00$, respectively

$m_i^z = m_i$). We have studied the influence of the coupling constant $R_{ab} = J_{ab}/J$ between the films, and the influence of the coupling constant $R_b = J_b/J$ of the material B on the magnetic behavior of the whole system. The effect of surface exchange coupling $R_s = J_s/J$ has been also investigated (we have set $R_1 = J_1/J$).

First we have studied the case of a simple film ($R_b = R_{ab} = 1.00$). We present the magnetization profiles in Fig. 2 for two surface exchange constants: $R_s = R_1 = 1.00$ and $R_s = R_1 = 1.50$. The critical temperature T_c increases with $R_s(T_c = 1.2522$ and $T_c = 1.4013$, respectively). In the first case the magnetization goes to higher value in the central atomic layer and decreases at the surface atomic layer. We have the contrary situation in the second case $R_s = 1.50$. In the films the surface magnetization maintains the same finite magnetization in the central atomic layers.

For a simple film (with two surfaces) there exists a critical value R_s^c of R_s depending on R_1 such that: when increasing R_1 the critical value R_s^c decreases. For $R_s < R_s^c(R_s > R_s^c)$ the critical temperature increases (decreases) with the increase of R_s.

For $R_a = R_1 = 1.00$ we get $R_s^c = 1.3069$, this is in good agreement with the values reported by Sarmento et al. [46] and Wiatrowski et al. [47].

In Fig. 3 we show magnetization profiles of the system for two different interface coupling constants R_{ab}, with a fixed ratio of the coupling exchange in material B: $R_b = 0.50$. The critical temperature in the second film B is therefore low compared to those of the two ferromagnetic films A. In Fig. 3a and b we present the calculated magnetization profiles for low $R_{ab} = 0.30$ and high $R_{ab} = 2.50$ coupling exchange interactions at the interface, respectively. The critical temperature increases with the ratio $R_{ab} : T_c = 1.1703$ (1.2301) for $R_{ab} = 0.30$ (2.50), respectively.

In the first case (Fig. 3a), due to the weak coupling between the films, the magnetization in material B breaks down for temperature around $T/T_c \approx 0.6$, while in the second case the magnetization goes to zero at $T/T_c \approx 0.80$. More interesting is the fact we observe in the weak coupling case: that the magnetization breaks down rapidly at the interface and takes the same value throughout the film B. We also notice in films A notable boundary effects from both surface layer and A–B interface where the magnetization is lower than in A ($i = 2$, 3 and 10, 11).

On the contrary in the high coupling case (Fig. 3b), we obtain now an increase of the magnetization at the A–B interface: the highest being in A; the lowest values of magnetization in B are obtained for the two central atomic layers ($i = 6$ and 7). Due to the

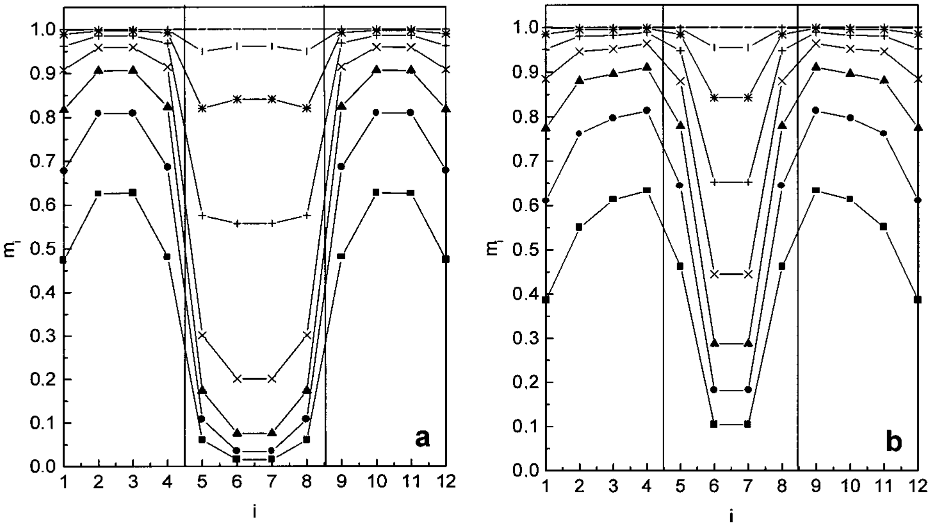

Fig. 3. The magnetization profile of the system for a) $R_b = 1.00$, $R_s = R_1 = 1.00$ and $R_{ab} = 0.30$, b) $R_b = 1.00$, $R_s = R_1 = 1.00$ and $R_{ab} = 2.50$, at different temperature $T/T_c = 0.9$ (■), 0.8 (●), 0.7 (▲), 0.6 (×), 0.5 (+), 0.4 (*), 0.3 (|), 0.1 (–)

effect of the ratio R_{ab}, the neighboring atomic layers ($i = 5$ and 8) have now a higher value of magnetization.

In Fig. 4a, we have plotted the magnetization profiles as a function of the interface exchange coupling R_{ab}, for a fixed value of $R_b = 1.00$ (i.e. identical materials in the

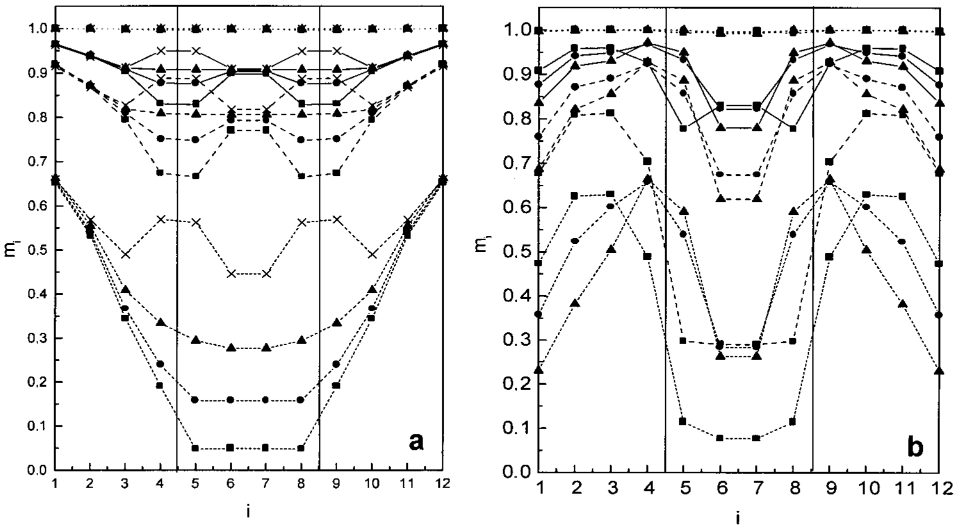

Fig. 4. The magnetization profile of the system for different interface coupling constant: a) $R_{ab} = 0.20$ (■), 0.60 (●), 1.00 (▲), and 2.00 (×), with $R_s = R_1 = 1.50$ and $R_b = 1.00$; b) $R_{ab} = 0.30$ (■), 2.00 (●), and 3.00 (▲), with $R_s = R_1 = 1.00$ and $R_b = 0.75$. Different ratios T/T_c: dotted (0.30), solid (0.60), dashed (0.70) and short-dashed (0.90) lines

three slabs) and $R_s = R_1 = 1.50$, and different values of the temperature: $T/T_c \geq 0.3$. For $R_{ab} = 0.20$ or 0.60 we find at the interface a strong lowering of the magnetization in A and B. We have similar values in the two materials. The highest value of the magnetization in film B is the central atomic layer magnetization. However, in material A the effect of the surface layer is dominant in atomic layers ($i = 1, 2, 11, 12$).

On the contrary, for $R_{ab} = 1.00$ or 2.00, the magnetization at the interface increases. The central layer magnetization in B is now lower than that at the interface. For $T/T_c = 0.9$ all magnetizations in the multilayer from surface to central atomic layers go to lesser values. Only for $R_{ab} = 2.00$ the profiles are the same for any values of the ratio T/T_c.

In Fig. 4b, we show the magnetization profiles as a function of the interface exchange coupling R_{ab}. The value of the exchange constant R_b in material B is lower than that in material A: $R_b = 0.75$ and $R_s = R_1 = 1.00$.

For $R_{ab} = 2.00$ or 3.00, in material A at the interface ($i = 4, 9$) the magnetization goes to higher values. We have a strong lowering of the magnetization in B, and the profile changes in comparison to Fig. 4a. Indeed, in material B, the highest values of the magnetization are obtained close to the interface.

For $R_{ab} = 0.30$ the magnetization at the interface is lower than in the preceding case ($R_{ab} = 2.0$ or 3.0). The values of the magnetization are higher in A. It decreases though the interface and reaches minimum values in B ($i = 4, 9$). Thus, for highest values of the ratio $T/T_c \geq 0.70$ the magnetization of central atomic layers in B is lesser than that of the whole system.

To summarize on the effect of the interface coupling exchange constant we note the existence of a critical value at R^c_{ab} of the ratio R_{ab} for which the critical value of temperature of the whole system is the bulk critical value of temperature of material A. This critical value depends on the value of R_b such that when $R_{ab} > R^c_{ab}$, the magnetization is stronger at the interface. The opposite situation is obtained when $R_{ab} < R^c_{ab}$. This suggests that there exists an interface magnetism in the system; indeed, for $R_{ab} > R^c_{ab}$ the system may order in the interface layers before it orders in the other layers.

In Fig. 5 we have plotted the magnetization profiles for different values of the ratio R_b. For a fixed value of $R_{ab} = 1.20$ and $R_s = R_1 = 1.00$, we have considered the effect of the weak exchange coupling constant R_b for $T/T_c \geq 0.6$. Due to the weak

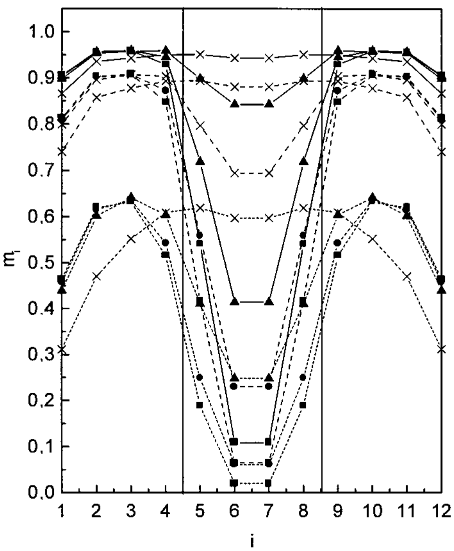

Fig. 5. The magnetization profile of the system for different exchange coupling constant in material B: $R_b = 0.30$ (■), 0.50 (●), 0.75 (▲) and 1.00 (×), with $R_s = R_1 = 1.00$ and $R_{ab} = 1.20$. Solid, dashed and short-dashed lines corresponding to the value of the ratio $T/T_c = 0.60, 0.70, 0.90$, respectively

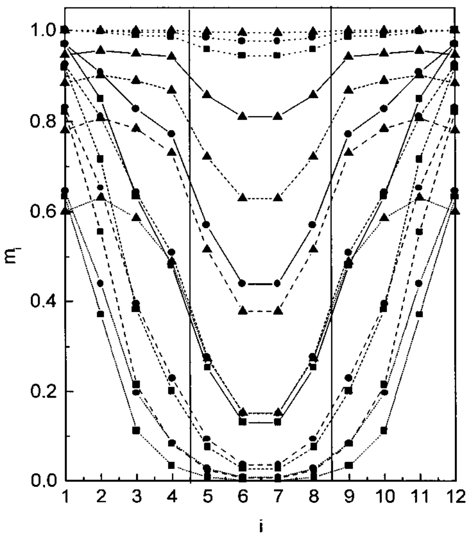

Fig. 6. The magnetization profile of the system for different surface exchange coupling constant: $R_s = 2.20$ (■), 1.8 (●), and 1.20 (▲), with $R_{ab} = 1.00$ and $R_b = 0.75$. Dotted, solid, dashed, short-dashed and short-dotted lines corresponding to the value of the ratio $T/T_c = 0.30$, 0.60, 0.70, 0.80, 0.90, respectively

coupling the magnetization breaks down in material B (case $R_b < 1$). The temperature dependence of the magnetization shows quite similar profiles for different ratios T/T_c. If $R_b = 1$ the magnetization of the whole system m_i is identical to that of a simple film. The critical temperature increases with the ratio R_b. Hence for $R_b = 0.30$, 0.50, 1.00, $T_c = 1.1773$, 1.18240, 1.27234, respectively.

The effect of the surface exchange coupling constant in a sandwich multilayer is shown in Fig. 6. For $R_b = 0.75$ and $R_{ab} = 1.00$, we have plotted the magnetization profiles for different values of $R_s = J_s/J(R_s = R_1)$. We can see that if T/T_c is around 0.30, the magnetization breaks down in B and is practically equal to unity in A. For $R_s = 2.20$ or 1.80 the magnetization in both materials breaks down and goes to zero at $T/T_c \approx 0.70$. The central atomic layer magnetization in the film goes to zero and attains the highest value for magnetization layers at the surface film. We have also plotted the magnetization profile in the case $R_s = 1.20$. The magnetization goes to a lesser value at the surface when T/T_c increases. At low temperature the magnetization goes to lower values in material B. In material A the lesser value is obtained at the interface.

The main important result in this section is that there exists a critical value R_s^c of R_s in the multilayer at which the critical value of the temperature is the bulk critical temperature in A, and which depends on R_b and R_{ab} (the dependence on R_{ab} is not investigated here). When $R_s > R_s^c$ the magnetization is higher at the surface ($i = 1$ and 12). When $R_s < R_s^c$ we have the opposite situation.

From a theoretical point of view, the magnetization profiles give very good insight into the magnetic behavior of the system. From an experimental point of view, however, the whole magnetization of the system is more interesting. We have calculated many dependencies of the magnetization $M(T)$ of the reduced interface coupling, the ratio of the exchange interaction in material B and the ratio of exchange interaction at the surface, respectively. However, we present only some particularly interesting results.

In Fig. 7 we have plotted the magnetization $M(T) = \frac{1}{12}\sum_{i=1}^{12} m_i$ as a function of temperature T/J, the ratios $R_b = 1.00$ and $R_s = R_1 = 1.50$ are fixed. The interface exchange ratio R_{ab} is varied. From comparison we have also plotted the magnetization in the case of a simple film (dashed curve) ($R_b = R_{ab} = R_s = R_1 = 1.00$). All the curves are situated above the dashed curve (simple film). It is clear from this figure that the

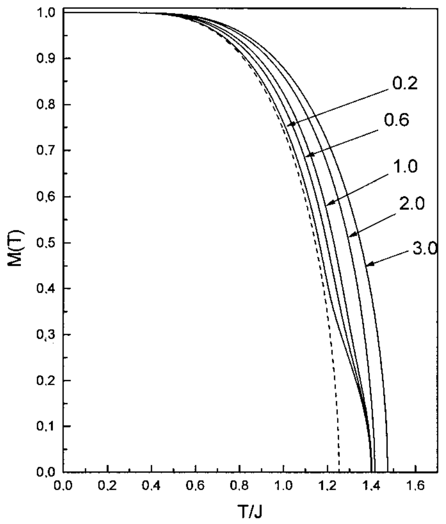

Fig. 7

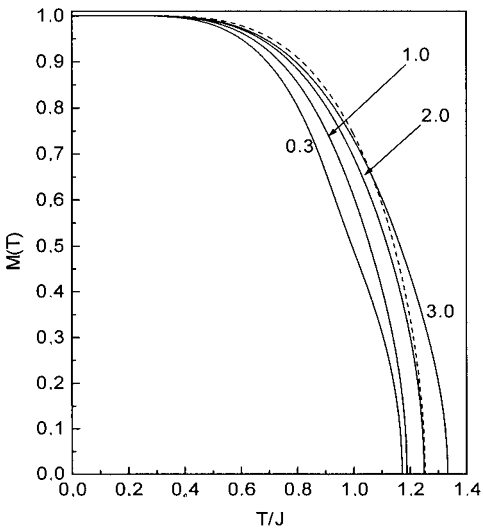

Fig. 8

Fig. 7. The variation of the magnetization $M(T)$ of the system versus T/J when $R_b = 1.00$ and $R_s = R_1 = 1.50$. The number accompanying each curve denotes the value of R_{ab}. Dashed curve is obtained of a simple thin film ($R_s = R_1 = 1.00$)

Fig. 8. The variation of the magnetization $M(T)$ of the system versus T/J when $R_b = 0.75$ and $R_s = R_1 = 1.00$. The number accompanying each curve denotes the value of R_{ab}. Dashed curve is obtained of a simple thin film ($R_s = R_1 = 1.00$)

magnetization decreases with the increase of the temperature T/J and vanishes at critical temperature of the multilayer. For small values of R_{ab}, the curves present a small inflexion and converge to a critical value T_c of temperature. However, they are sensitive to strength of R_{ab} and separate for large values of R_{ab}.

In Fig. 8 we show the variation of M as function of temperature T/J for different values of R_{ab}. The ratios $R_b = 0.75$ and $R_s = R_1 = 1.00$ are fixed. At low temperature the magnetization curves decrease and are below the curve of a simple film (dashed curve). For large values of R_{ab} (3.00) the magnetization curve crosses the magnetization curve of a simple film (dashed curve).

In Fig. 9 we show the variation of $M(T)$ as a function of the temperature T/J for several values of R_b while $R_{ab} = 1.20$ and $R_s = R_1 = 1.00$ remain fixed. We have a strong influence at low temperatures. The magnetization decreases when increasing the temperature and vanishes at the multilayer critical temperature. At small values of the ratio R_b (0.30 or 0.50) we found a large curvature of the magnetization curve that disappears when R_b approaches 1.00 and when T approaches T_c.

Figure 10 shows the variation of $M(T)$ as a function of T/J where the coupling constant $R_s = R_1$ is varied. $R_b = 0.75$ and $R_{ab} = 1.00$ are fixed. The critical temperature value increases with the increase of the ratio R_s. For $R_s = 1.80$ (2.20) we find $T_c = 1.6082$ (1.9089), respectively. We found a large increase of curvature for the magnetization $M(T)$ when increasing R_s. All curves present a large curvature near the value of the critical temperature. At higher value of $R_s (> R_s^c)$ the influence of the surface exchange is very large and therefore T_c is higher than that of a thin film (dashed curve).

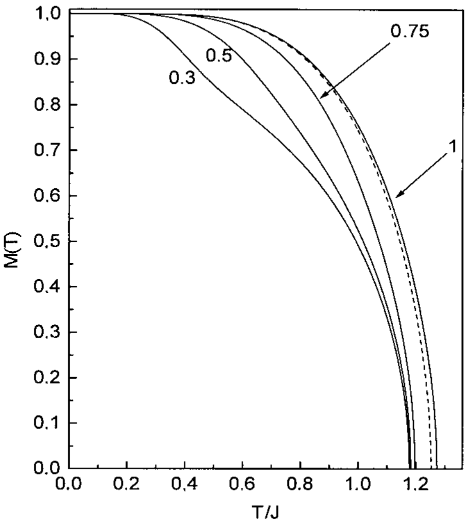

Fig. 9

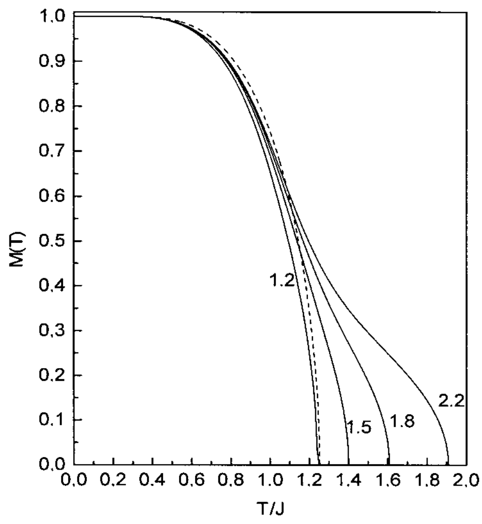

Fig. 10

Fig. 9. Magnetization $M(T)$ of the system versus T/J when $R_{ab} = 1.20$ and $R_s = R_1 = 1.00$. The number accompanying each curve denotes the value of R_b. Dashed curve is obtained of a simple thin film ($R_s = R_1 = 1.00$)

Fig. 10. Magnetization $M(T)$ of the system versus T/J when $R_b = 0.75$ and $R_{ab} = 1.00$. The number accompanying each curve denotes the value of $R_s (R_s = R_1)$. Dashed curve is obtained of a simple thin film ($R_s = R_1 = 1.00$)

4. Conclusion

In this work, we have shown that we can describe qualitatively the behavior of sandwich ferromagnetic multilayers with ferromagnetic coupling at the interfaces using the effective field theory in a spin-1/2 Ising model. The formalism of the transition temperature derivation obtained above is universal and can be used to study multilayers of various thicknesses and structures. Although we have considered a film with only twelve atomic ferromagnetic layers (all $J_{i,j} > 0$), the formulation is also applicable for antiferromagnetic couplings (some or all $J_{i,j} < 0$ in the whole system). We have also shown that we can give a description of the magnetic behavior of ferromagnetic coupling at the interface with the main important result: for each value of R_b there exists a critical value of the ratio R_{ab} such that when $R_{ab} < R^c_{ab} (R_{ab} > R^c_{ab})$ the magnetization decreases (increases), indeed due to the interface interlayer ordering dominates. Finally we have shown that the different curvatures of the $M(T)$ curves depend strongly on the different coupling constants R_{ab}, R_b or R_s. The critical temperature of the multilayers increases with the increase of R_{ab} and moves to higher values if R_b increases.

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