Journal of Magnetism and Magnetic Materials 213 (2000) 340–348

Journal of magnetism and magnetic materials

www.elsevier.com/locate/jmmm

Magnetic interlayer coupling in multilayers of fractional dimensionality

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Received 1 April 1999; received in revised form 5 January 2000

Abstract

Within analytical method we calculate the RKKY interaction between localized magnetic moments for a system of fractional (nonintegral) dimension. We provide the exact derivation of the spatial dependence of the RKKY exchange integral as an analytical function of dimensionality. Moreover, with the help of fractional analysis, we derive formulae for interlayer coupling in fractional multilayers. On the basis of the results obtained possibility of controlled interlayer interaction is shown.

PACS: 75.10. − b; 75.30.Hx; 75.70.Ak

Keywords: Magnetic superlattice; RKKY interaction; Fractional dimensionality; Interlayer coupling; Interface roughness

1. Introduction

The concept of fractional dimensionality has proved very successful in advancing our understanding, of how geometry affects the physical properties of the system [1]. Many laminar systems like Ag/Cu(0 0 1) overlayer or GaAs/Al_xGa_{1-x}As quantum wells and/or superlattices, as the layer thickness decreases, show dimensional crossover from 3D to almost 2D behavior (see Refs. [2–4] and references therein). Generally, the dimension of these systems changes with the monolayer coverage, wire thickness or temperature. In the case of rough interfaces, a nonintegral dimension of the stratified system can be interpreted in terms of fractal geometry (Haussdorff dimension [5]). However, some systems not having fractal structure also exhibit nonintegral dimensionality. Indeed, numerous physical problems involve basic objects, which are usually described by shrinking or stretching the shape of some characteristic functions. Invoking a fractional-dimensional interaction space in description of such a system offers a convenient alternative to long computational times [4]. In this case, a single parameter – the dimensionality – contains all of the information about the perturbation. This dimensionality may have several meanings. It may describe the number of coordinates to be dealt with, e.g. in problems with several particles. In this work, we shall be interested in another definition of the nonintegral dimensionality, considering it as a description of the anisotropy of a given medium [2–4]. We adopt the approach by He [2,3], who has shown that the anisotropic interactions in 3D

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PII: S0304-8853(00)00012-3

space become isotropic ones in lower fractional dimensional space, where the dimension is the Hausdorff dimension and is determined by the degree of anisotropy. Thus, in our case we assume that the fractional dimension arises due to the restraint conditions imposed (by interface potentials) on the motion of mobile particles.

The aim of this paper is to study the indirect magnetic interactions in magnetic multilayers, which show nonintegral dimensionality. First of all, we will calculate the exchange integrals of the indirect magnetic coupling (RKKY), between magnetic ions in a metallic system of nonintegral dimension. Having that, we will find expressions for the interlayer coupling between two ferromagnetic layers across metallic αD, nonmagnetic spacer. Two cases of fractal (e.g. the famous YBCO family [6]) and nonfractal interfaces will be considered.

2. RKKY interaction

At the advent of the RKKY theory the formulae for the exchange integrals in the case of 3D [7], 1D [8] and 2D [9] systems have been found. Very recently, a unified derivation of the RKKY exchange integral for any positive, integral dimension has been found [10]. Moreover, there is derived an asymptotic expression for the RKKY exchange integral in a system of fractional dimensionality. The $(2+\varepsilon)$D system is assumed as a 3D solid, in which the free electron system is described by anisotropic dispersion of the form

$$\varepsilon_k = (k_x^2 + k_y^2)/2 + \xi \cos(k_z z), \tag{1}$$

which is characteristic for periodic, layered electron gas systems with weak interlayer tunnelling [11]. Later, within perturbational approach, the RKKY exchange integral in the limit of large in-plane distances ($\boldsymbol{\rho} \to \infty$, $\boldsymbol{R} = \boldsymbol{\rho} + \boldsymbol{z}$) is derived. However, in any real magnetic system the magnetic order is governed by the coupling of nearest magnetic moments, thus the limit $\boldsymbol{\rho} \to \infty$ cannot give important information about magnetic system. In the paper [12], we have found the other asymptotic limit ($\boldsymbol{\rho} \to 0$) of the RKKY exchange in a system with free-electron energy spectrum given by Eq. (1). The expression for the RKKY exchange integral, obtained within perturbational approach reproduces correctly the 'z' dependence (along anisotropy axis), but shows spurious $\ln|\rho|$ divergence. Fortunately, this shortcoming can be easily removed, when correction of the type proposed by Nagaev and Podel'schikov [13] is performed. However, within an approach that is based on electron spectrum (1) analytical formula for the RKKY exchange, valid for any separation of interacting moments, cannot be obtained. This means that another approach to anisotropic free-electron systems is needed.

To calculate the RKKY exchange interaction in an anisotropic system we will apply a different approach presented in Refs. [2,3]. The anisotropic interactions in 3D space become isotropic ones in lower fractional dimensional space, where the dimension is the Hausdorff dimension and is determined by the degree of anisotropy [2,3]. Since the perturbational approach to the RKKY interaction involves integration over dynamical states of the free electrons, the fact that the space is isotropic (though αD) offers evident calculational advantages.

In the following generalizing approach by Aristov [10], we will give an exact formula for the RKKY range function for any system, which exhibits nonintegral dimension α, with $2\mathrm{D} < \alpha\mathrm{D} < 3\mathrm{D}$.

The starting point for any description of metallic magnetic systems is the case of dilute alloys, when a few TM or RE ions are immersed in the sea of host conduction electrons. The effective interaction between RE or TM localized moments is mediated via the free electrons. Within perturbative aproach, the RKKY interaction between magnetic moments of the magnetic ions (μ_i and μ_j) can be written as [7]

$$H(R_{ij}) = \tfrac{1}{2}A^2\chi(R_{ij})\boldsymbol{\mu}_i\boldsymbol{\mu}_j, \tag{2}$$

where $\chi(r_{ij})$ is the nonuniform static susceptibility. The explicit form of the $\chi(r_{ij})$ is given by [10]

$$\chi(R) = -T\sum_l G(\mathrm{i}\omega_l, R)^2, \tag{3}$$

where $\omega_l = \pi T(2l+1)$ are the Matsubara frequencies and the electronic Green's function is

$$G(i\omega, \boldsymbol{R}) = \int \frac{d^\alpha \boldsymbol{k}}{(2\pi)^\alpha} \cdot \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{R}}}{i\omega - \varepsilon_k}. \quad (4)$$

3. Electron gas of nonintegral dimensionality

Extension of Eq. (4) on αD-space requires calculation of the integrals $\int d^\alpha k \ldots$ over the αD-space (with $2 < \alpha < 3$) of dynamical states of the free electrons. In this case, there arises a question, whether the k-space description is valid in the αD system. Since in laminar systems the translational symmetry is broken, one would expect that the proper answer is no, on the other hand, the existence of band structure in liquid metals and random alloys suggests that a positive answer is possible. This is given by Tsallis and Maynard [14], who have shown, that the k-space formalism can be applied to the description of dynamical states in the fractal system, when only statistical invariance under translation is observed. The metallic superlattices possess crystalline in-plane symmetry and the fractional dimensionality arises not from intrinsic disorder, but due to the restraints imposed onto motion of free particles. The magnetic interlayer coupling in superlatices can be well described in terms of k-vectors spanning the Fermi surface of the spacer layer. This gives experimental evidence of the applicability of k-space formalism in these systems and indicates that the arguments of Tsallis and Maynard [14] still hold. The direct observation of band structure in a Ag/Cu(0 0 1) overlayer [15] confirms the applicability of k-space formalism in layered systems. Moreover, the observed gradual change of the valence-band spectra, from 2D into bulk 3D at higher coverages justifies fractional interpolation between 2D and 3D dimensionalities.

Concerning the problem of mobile particle confined within a layer the question arises, what is the spatial dimension α, which measures the anisotropy of the system. A possible answer in the case of superlattice is given in Ref. [16], where the FD is defined as $\alpha = 2 + \gamma = 2 + \mu_0/\mu_z$ where μ_0 and μ_z are the on-axis reduced effective masses in the 3D crystal and in the superlattice, respectively. Another possible choice is to express α in terms of the effective quantum well width L_w^* in the case of excitons having the extension ξ is given by the expresion $\alpha = 3 - e^{-L_w^*/\xi}$ [4].

Below we will give an illustrative example, that shows in which way the FD can arise in laminar systems. Movement of mobile electrons/holes is confined within a layer of thickness 2a by (generally not rectangular) potential barrier. Let us assume, that the potential $V(\boldsymbol{r}), (\boldsymbol{r} = \boldsymbol{\rho} + z)$ which is the source of anisotropy has the following form

$$V(\boldsymbol{r}) = \begin{cases} 0 & z \in (-a, a), \\ V(|z|), & z \notin (-a, a). \end{cases} \quad (5)$$

It is natural to expect, that the eigenstates of the mobile electrons confined within the quantum-well defined by potential (5) are given by

$$\psi_n(\boldsymbol{r}) = \begin{cases} A \cdot e^{i\boldsymbol{k}_n \cdot \boldsymbol{r}}, & z \in (-a, a) \\ B\, e^{ik_{n_x}x + ik_{n_y}y} \phi_n(z)\, e^{-\kappa \cdot |z|} & z \notin (-a, a), \end{cases} \quad (6)$$

where n_p are integers that label the electron eigenstates within the quantum well. The eigenstates are the Bloch waves within the $z \in (-a, a)$ region and the envelope functions of the nth electron (hole) quantum level shows exponential decay outside. Provided that the Hamiltonian of the system has the form $\hat{H} = \hat{T} + V(\boldsymbol{r})$, where $\hat{T}$-is the kinetic energy and $V(r)$ the confinement potential the energy spectrum of the free particles within the quantum well is given by $\varepsilon_k = \hbar^2 k^2/(2m)$, where k is a wave vector.

If the potential $V(r)$ is known, accounting for the boundary conditions, we can find the confined levels E_n in the quantum well. However, let us consider the inverse problem. Suppose that the allowed energy levels behave as

$$E_n = A \cdot n^{2\xi}, \quad (7)$$

where n is an integer and $\xi > 1$. Evidently, with the eigenvalues E_n and eigenfunctions ψ_n (see Eq. (6)) known, the potential $V(r)$ is fully determined. Indeed, we can write

$$V(r) = \sum_{m,n} [E_n \cdot \delta_{m,n} - \langle \psi_n | \hat{T} | \psi_m \rangle] \cdot \psi_n(r). \quad (8)$$

According to Eq. (6) the dynamical states within the layer can be labeled by the wave vectors $\boldsymbol{k}$. However, contrary to the situation observed in the conventional 3D (translationally invariant) system the density of states within the k-space is not uniform. Indeed, let us denote by $N(V_{\mathrm{p}})$ the number of allowed k_n states that fill the volume of parallelopiped V_{p}. Suppose that any length scale within the k-space is changed as $\boldsymbol{k} \to \lambda \cdot \boldsymbol{k}$, evidently $V_{\mathrm{p}} \to V_{\mathrm{p}}^{\lambda}$. Then, in view of Eq. (7) the number of states $N(V_{\mathrm{p}})$ scales as $N(V_{\mathrm{p}}^{\lambda}) = \lambda^{2+\eta} \cdot N(V_{\mathrm{p}})$, where $\eta = 1/\xi$. This relation is characteristic of a system which shows fractional (spectral) dimensionality $\alpha = 2 + \eta$ [14]. This means that the layered dynamical system described by relations (5)–(8) becomes uniformly dense (isotropic) when treated within FD space with (spectral) dimension $\alpha = 2 + \eta$. The example presented above shows the meaning of the FD we shall be using further on. However, we do not assume that the arguments presented above are the main justification to apply the idea of FD in the layered systems. For that, the experimental evidence of FD behavior of mobile charge carriers within laminar systems is cruicial. Systems like Ag/Cu(0 0 1) overlayer [15] or $\mathrm{GaAs/Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ quantum wells and/or superlattices, as the layer thickness decreases, show dimensional crossover from 3D to almost 2D behavior (see Refs. [2,3] references therein).

4. RKKY interaction in a system of fractional dimensionality

Another question that arises with the application of Eq. (4) is the problem of integration over the αD space. The mathematical basis for such a calculation was derived by Stillinger [17] and we will apply this formalism further on.

In the case of integral dimension $\alpha = n$, the integration over $\boldsymbol{k}$ reduces to subsequent (iterative) integration over Cartesian components of this vector (see Ref. [10]). However, in arbitrary dimension such a procedure cannot be performed. To circumvent this problem, we will apply the method by Stillinger [17], who devised an equivalent formula, that reduces integration in αD to integration in 2D system

$$\int_{\alpha D} \mathrm{d}^{\alpha}\boldsymbol{r} = \frac{1}{\Gamma((\alpha-1)/2)} \cdot 2\pi^{(\alpha-1)/2} \int r^{\alpha-1}\,\mathrm{d}r \times \int_0^{\pi} \sin^{\alpha-2}\theta\,\mathrm{d}\theta, \tag{9}$$

where $\Gamma(x)$ is the Euler gamma function. It is important to note, that in the αD an equivalent of the Laplace operator has the form [17]

$$\nabla^2\Psi = \left[\frac{\partial^2}{\partial r^2} + \frac{\alpha-1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2\sin^{\alpha-2}\theta}\frac{\partial}{\partial\theta}\sin^{\alpha-2}\theta\frac{\partial}{\partial\theta}\right]\Psi, \tag{10}$$

thus the wave function of the free electrons still has the $\exp(\mathrm{i}\boldsymbol{kr}) = \exp(\mathrm{i}kr\cos\theta)$ form. Moreover, the free-electron spectrum is again $\varepsilon_k = k^2/(2m) - \mu$.

Contrary to the approach by Aristov [10] the integral in Eq. (4) is carried out with the use of formula (9), which allows us to integrate over a space of arbitrary dimension. We will focus our attention on the low-temperature limit and then make use of the relation $T\sum_l \to \int_{-\infty}^{\infty} \mathrm{d}\omega/2\pi$. Making use of Eq. (9), we can rewrite Eq. (4) as.

$$G(\mathrm{i}\omega, \boldsymbol{R}) = A\int \frac{k^{\alpha-1}\,\mathrm{d}k}{(2\pi)^{\alpha}} \int_0^{\pi} \mathrm{d}\theta\,\sin^{\alpha-2}\theta \times \frac{\mathrm{e}^{\mathrm{i}kR\cos\theta}}{(z-(\hbar^2k^2/2m))}, \tag{11}$$

where $z = \mathrm{i}\omega + \mu$. The integration over θ can be performed, if we recall the definition of the Bessel functions $J_\nu(z)$ ([18, p. 54]. Inserting this into Eq. (11) we can write

$$G(\mathrm{i}\omega, \boldsymbol{R}) = \frac{A'}{R^{\nu}} \int_0^{\infty} \frac{k^{\nu+1}J_\nu(kR)}{k^2+\beta^2}\,\mathrm{d}k \tag{12}$$

with $\beta^2 = -2mz$ and $\nu = (\alpha-2)/2$. By virtue of the identity [18, p. 110, formula 58], valid for $a > 0$, $\operatorname{Re}\beta > 0$ and $-1 < \operatorname{Re}\nu < \frac{3}{2}$), we find that integration in Eq. (11) over k gives us $G(\mathrm{i}\omega, R)$ in the following form

$$G(\mathrm{i}\omega, \boldsymbol{R}) = D\cdot\left(\frac{\sqrt{-2z\rho}}{\rho}\right)^{\nu} \cdot K_\nu(\sqrt{-2z\rho}), \tag{13}$$

where $K_\nu(x)$ is the McDonald function of order ν, $\rho = mR^2$ and D is a constant. In the integration, the root of $\sqrt{-2z\rho}$ should be chosen from the condition of its positive real part. Contour integration over ω in Eq. (11), accounting for the discontinuity of $G(\mathrm{i}\omega, R)$ at $\omega = 0$ [10] leads to an expression for the RKKY exchange integral in an αD system (compare Ref. [10])

$$\chi(r) \approx \frac{\chi_0}{r^\alpha} r^2 [J_{\alpha/2-1}(x) Y_{\alpha/2-1}(x) + J_{\alpha/2}(x) Y_{\alpha/2}(x)]. \tag{14}$$

with $Y_\nu(x)$ being the Neumann function [18]. Formula (14) has the same structure as the result obtained by Aristov [10] in the case of a system of integral dimensionality. The calculated static susceptibility that determines exchange integral $J(r) = J_0 \cdot \chi(r)$ of the RKKY interaction (2) in αD, shows conventional, sign-reversal oscillatory behavior with the period governed by the wave vector $2k_\mathrm{F}$. It is interesting, that the result obtained (14) is an analytical function of the dimension α. One should note, that result (14) is valid also in the case of fractal overlayers, both spontaneously grown or fabricated by ionic beam bombardment [19].

5. Interlayer coupling

Conventional heteroepitaxial magnetic superlattice consists of alternating layers of elemental constituents. One of them is nonmagnetic metal, while the other is a ferromagnet with in-layer ordering. The most unusual effect in magnetic superlattices is the observation, that the RKKY-like, magnetic coupling across the nonmagnetic spacer layer is an oscillatory function of the spacer thickness, with a surprisingly long oscillation period. The properties of the interlayer coupling are due to the quantum confinement of the free electrons within spacer well [20]. In the case when the magnetic layers are nonmetallic, one would expect that the density of electron gas has a layered structure and is extremely anisotropic. In principle, the energy spectrum of the free electrons can be descibed by Eq. (1). As we have discussed above the anisotropy of the free electron spectrum can be taken into account by the assumption, that the electron gas is described by a lower (Haussdorff) dimension [2,3]. Below, we will assume that in a layered free-electron system the restraints imposed (by the interface potentials) on the motion of free particles may result in the αD behavior (nonintegral spectral dimension). Basing on this assumption we will study the problem of interlayer magnetic coupling in superlattices or multilayers, which is mediated via free particles within αD spacer. In the following, we will derive an expression for the interlayer coupling constant in the case of αD system. Result (14) opens the possibility to study magnetic interlayer coupling in such systems.

Having calculated the coupling of two magnetic moments in the αD metallic system, we can obtain the effective interplane coupling in the magnetic superlattice. We will follow the approach presented by Yafet [21] or Bruno and Chappert [22]. Let us consider a system of two magnetic planes $F1$ and $F2$, separated by a metallic αD spacer of thickness d. We assume that planes (e.g. $z = 0$ and a planes in a simple cubic lattice) are populated with ferromagnetically ordered magnetic moments.

The magnetic interlayer coupling is obtained from Eq. (14) by summing $H(R_{ij})$ over all pairs ij, i and j running, respectively, on $F1$ and $F2$. The coupling per unit area can be written [22] as

$$E_{1,2} = I_{1,2} \cos \theta_{1,2}, \tag{15}$$

where $\theta_{1,2}$ is the angle between the magnetizations of $F1$ and $F2$. The interlayer coupling constant $I_{1,2}$ is given by [22]

$$I_{1,2} = J_0 \frac{d}{V_0} S^2 \sum_{j \varepsilon F2} \chi(R_{Oj}) = I_0 \sum_{j \varepsilon F2} \chi(R_{Oj}), \tag{16}$$

where O labels one site of $F1$ taken as the origin. In the continuum limit of the moment distribution, the range function $I_{1,2}$ of the interplane coupling becomes independent of the variables x and y and we perform in Eq. (16) the substitution

$$\sum_{F2} \to \frac{d}{V_0} \int_{F2} d^2 R_{\|} = \frac{d}{V_0} \int_{F2} \rho \, \mathrm{d}\rho = \frac{d}{V_0} \int_{F2} r \, \mathrm{d}r. \tag{17}$$

where $r\,\mathrm{d}r = \rho\,\mathrm{d}\rho$ with $\rho^2 = x^2 + y^2$. If the z-axis is chosen to go through observation point r, then the range function $I_{1,2}$ is given by the following integral over the source points of the αD range function (14)

$$I_{1,2} = I_0 \int_d^\infty r^{3-\alpha}[J_{\alpha/2-1}(k_F r) Y_{\alpha/2-1}(k_F r) + J_{\alpha/2}(k_F r) Y_{\alpha/2}(k_F r)]\, dr. \quad (18)$$

If we make use of the Bessel function identities [18, p. 111, Eq. (65) and p. 20, Eq. (56)] we arrive at

$$I_{1,2} = I_0 \int_0^\infty dt \int_{k_F d}^\infty x^{2-\alpha} J_{\alpha-1}(2x \operatorname{ch} t)\, dx, \quad (19)$$

where $x = k_F r$. With the use of relation [18, p. 55, Eq. (2)], the integration over x can be easily performed giving us

$$I_{1,2} = I_0 \int_0^\infty dt (2k_F d \operatorname{ch} t)^{-2} J_{\alpha-2}(2k_F d \operatorname{ch} t). \quad (20)$$

Using relations [18, p. 111, Eq. (65) and p. 20, Eq. (56)] backwards, we can perform integration in Eq. (18) and obtain the spatial dependence of the interplane coupling

$$I_{1,2}(d) = I_0 \frac{d^2}{d^{\alpha-2}}[J_{\alpha/2-2}(k_F d) Y_{\alpha/2-2}(k_F d) + 2J_{\alpha/2-1}(k_F d) Y_{\alpha/2-1}(k_F d) + J_{\alpha/2}(k_F d) Y_{\alpha/2}(k_F d)]. \quad (21)$$

The effective interlayer coupling (21), between magnetic layers across αD metallic spacer is represented by the RKKY – reminiscent exchange integral in the $(\alpha - 1)D$ space. Result (21) can explain experimental data in the Eu/Se superlattice, in which the interlayer coupling falls off as $I_{1,2}(d) \approx d^{-1.1}$ [23]. This is clear evidence of the fractional dimensionality of the spacer, since in the case of 3D systems there should be $I_{1,2} \approx d^{-2}$ [22]. Similarly to expression (14), interplane coupling (21) shows oscillatory behavior governed by the $2k_F$ wave vector. The characteristic features of the interplane coupling, such as occurrence of long periods as well as multiperiodic oscillations can be explained when discrete structure of the superlattice is considered [22]. In our calculations we have assumed that the in-layer magnetization is constant. However, the in-layer magnetic ordering is not neccesarily uniform, neither in superlattices [24] in the case of overlayers [25]. In this case result (21) does not hold.

6. Roughness of the interfaces

Result (21) was obtained under the assumption that the interfaces are ideally flat. However, the interfaces can exhibit roughness, which can modify the interlayer coupling. Till now, only the special case of correlated steps on a surface has been considered [26]. In the following, we will present an alternative approach based on the concept of fractals. The surface/interface roughness often exhibits a self-affine structure [1,27] and different scaling behavior can be found as a function of thickness and lateral length scale L. In this case, the interface is characterized by the mean-square average roughness (height-correlation function) $\xi(L)$.

$$\xi(L) = \left[\frac{1}{L}\sum_j (z_j - \varrho)^2\right]^{1/2}, \quad (22)$$

and the scaling of the roughness parameter is given by $\xi(L) \approx L^\beta$, with β being a fraction.

In our approach, we will consider a trilayer in which the outer ferromagnetic layers are separated by a nonmagnetic of average thickness ϱ. We will assume that at least one of the interfaces (F_1^β or F_2^β) is self-similar and its dimension equals $2 + \beta$ (with $0 < \beta < 1$). As usualy, we assume that the interlayer coupling between layers is mediated by the free charge carriers of the central layer. The restraint conditions imposed (by the interface potentials) on the motion of free particles cause the k-space of their eigenstates to show fractional (spectral) dimensionality. This means that the magnetic interaction between two ionic moments, that belong to different magnetic layers is described by formula (14).

Similarly, as in the case of ideally flat interfaces, the magnetic interlayer coupling can be obtained by summing contributions from all pairs of moments $\boldsymbol{m}_i$ and $\boldsymbol{m}_j$ with i and j running over F_1^β and F_2^β. The interlayer coupling energy $E_{1,2}$ per unit measure of the interface can be expressed by the formula $E_{1,2} = I_{1,2}^{\alpha,\beta} \cos \Theta_{1,2}$ (compare formula

(17)). The interlayer exchange coupling integral $I^{\alpha,\beta}_{1,2}$ is thus given by

$$I^{\alpha,\beta}_{1,2} = J_0 \sum_{j\varepsilon F_2^\beta} \chi_\alpha(R_{0j}). \tag{23}$$

In the continuum limit we make use of Eq. (9), which alows us to integrate over $\sigma \in F_2^\beta$. In view of Eq. (22) we find, that volume element of F_2^β behaves as $\mathrm{d}V_2^\beta = |\boldsymbol{r}-\boldsymbol{\varrho}|^{1+\beta}\,\mathrm{d}|\boldsymbol{r}-\boldsymbol{\varrho}|$. Thus, Eq. (23) can be rewritten as

$$\sum_{j\varepsilon F_2^\beta} \chi_\alpha(R_{0j}) \to C\int_{F_2^\beta} |\boldsymbol{r}-\boldsymbol{\varrho}|^{1+\beta}\chi(k_\mathrm{F} r)\,\mathrm{d}|\boldsymbol{r}-\boldsymbol{\varrho}|. \tag{24}$$

The term $|\boldsymbol{r}-\boldsymbol{\varrho}|^{1+\beta}$ can be expanded in power series of $|r|$ and $|\varrho|$. Since β is a fraction we should make use of the fractional version of the Taylor formula [28]

$$f(x) = \sum_{j=0}^{n-1} \frac{(D^{\alpha+j}f)(a)}{\Gamma(\alpha+j+1)}(x-a)^{\alpha+j} + R_n(x). \tag{25}$$

The symbol $(D^{\alpha+j}f)(x)$ denotes the derivative of fractional order $j+\alpha$ of the real function $f(x)$. The fractional calculus is a powerful tool in theoretical studies of systems which show fractional dimensionality [2–4,29,30]. The Riemann–Louville diffintegral DI^α is defined as follows [28,31,32]

$$(\mathrm{DI}^\alpha f)(x) = \frac{1}{\Gamma(\alpha-1)}\int_0^\infty (x-t)^{\alpha-1} f(t)\,\mathrm{d}t, \tag{26}$$

and is a fractional counterpart of derivative of fractional order $D^\alpha = \mathrm{d}^\alpha/\mathrm{d}x^\alpha$ (for $\alpha<0$) or fractional integral I^α (for $\alpha>0$).

For the case of large separations the leading term of the interplane coupling Eqs. (23) and (24), can be calculated if we limit ourselves to the first term of expansion i.e. $|\boldsymbol{r}-\boldsymbol{\varrho}|^{1+\beta} \approx |r|^{1+\beta}$. Thus, if we make use of the identities [18, p. 111, Eq. (65) and p. 20, Eq. (56)], the interlayer exchange integral (23) and (24) reads

$$I^{\alpha\beta}_{1,2} = I_0 \int_0^\infty \mathrm{d}t \int_\varrho^\infty r^{2-\alpha+\beta} J_{\alpha-1}(2k_\mathrm{F} r\,\mathrm{ch}\,t)\,\mathrm{d}r. \tag{27}$$

Integral (27) can be calculated strictly only for some values of α and β. In the case of arbitrary α and β only approximate formulae for the interlayer coupling parameter $I^{\alpha,\beta}_{1,2}$ can be obtained. To calculate $I^{\alpha,\beta}_{1,2}$ let us recall the identity fulfilled by fractional diffintegrals of the Bessel functions [28, p. 48]:

$$(\mathrm{DI}^\lambda_{0^+})[x^{\mu/2}J_\mu(\sqrt{x})] = 2^\lambda x^{(\mu+\lambda)/2} J_{\mu+\lambda}(\sqrt{x}). \tag{28}$$

Having identity (28) we can integrate over ϱ in Eq. (27) using the formula for fractional diffintegration by parts [28, p. 42]

$$\int_a^b \phi(x) I^\alpha_{a^+}\psi(x)\,\mathrm{d}x = \int_a^b \psi(x) I^\alpha_{b^-}\phi(x)\,\mathrm{d}x. \tag{29}$$

If we account that $I^\lambda_{0^+}x^\mu \sim x^{\mu+\lambda}$ [28, p. 140, Eq. (27)] can be reduced to

$$I^{\alpha\beta}_{1,2} = I_0(\varrho)^{\mu+1}\int_0^\infty (\mathrm{ch}\,t)^{-2-\beta/2+\alpha} J_\mu(2k_\mathrm{F}\varrho\,\mathrm{ch}\,t)\,\mathrm{d}t. \tag{30}$$

In the case of arbitrary α and β the integration over variable t in Eq. (30) cannot be performed in a direct way. However, if we have $\alpha = 2+\beta/2$. Eq. (30) takes the following form:

$$I^{\alpha\beta}_{1,2} = I_0(\varrho)^{\nu+1} J_{\nu/2}(k_\mathrm{F}\varrho) Y_{\nu/2}(k_\mathrm{F}\varrho), \tag{31}$$

where $\nu = 1+\beta/2$. The condition $\alpha = 2+\beta/2$ seems to be very restrictive. However, some layered systems exhibit continuous dimensional crossover, when external conditions (e.g. temperature or magnetic field) are changed. This means that in such systems this peculiar condition can always be fulfilled. In the case of arbitrary α and β the integration over t in Eq. (30) cannot be performed directly. Fortunately, with the help of fractional analysis, we can transform integral (27) to a more simple form, which allows us to draw some conclusions concerning the interlayer coupling. Setting $x = (2k_\mathrm{F}\varrho\,\mathrm{ch}\,t)^2$ and using the identity

$$\frac{\mathrm{d}^\lambda}{\mathrm{d}x^\lambda} = (2k_\mathrm{F}\varrho\,\mathrm{ch}\,t)^{-2\lambda}\frac{\mathrm{d}^\lambda}{\mathrm{d}(k_\mathrm{F}^2)^\lambda}, \tag{32}$$

which results from the definition of fractional derivatives, we can rewrite Eq. (30) in the following form:

$$I^{\alpha\beta}_{1,2} = I_0(\varrho)^{\alpha-1}(k_\mathrm{F})^{-\nu}\int_0^\infty \mathrm{d}t\frac{\mathrm{d}^\lambda}{\mathrm{d}(k_\mathrm{F}^2)^\lambda}((k_\mathrm{F})^\nu J_\nu(2k_\mathrm{F}\varrho\,\mathrm{ch}\,t)). \tag{33}$$

If we change order of differentiation and integration, then the integration over t leads to

$$I^{\alpha\beta}_{1,2} = J_0 \cdot (\varrho)^{\alpha-1} \frac{\mathrm{d}^{\lambda}}{\mathrm{d}(k_{\mathrm{F}}^2)^{\lambda}} [(k_{\mathrm{F}})^{\nu} J_{\nu/2}(k_{\mathrm{F}}\varrho) Y_{\nu/2}(k_{\mathrm{F}}\varrho)]$$

with $\lambda = \alpha - 2 - \beta/2$ and $\nu = 1 + \beta/2$.

Result (34) represents the leading term of the interlayer coupling constant (i.e. the term which dominates at large ϱ). The other term of expansion (24)–(25) can be calculated in a similar way as result (34). However, since we have assumed that the interface F^{β}_2 is self-affine it is evident that our calculations are valid for the superlattices with relatively thick spacer layers. In this case it suffices to study the properties of the leading term (34). Both expressions (31) and (34) show oscillatory behavior determined by the oscillations of the Bessel functions $J_{\nu/2}(k_{\mathrm{F}}\varrho)$ and $Y_{\nu/2}(k_{\mathrm{F}}\varrho)$. Similarly, as in the case of systems with integral dimension, the oscillation period is directly related to the $2k_{\mathrm{F}}$ wave vector.

The fact that expressions (21), (31) and (34) are analytical functions of α and β allows us to discuss the effect of dimensionality on the interlayer coupling. Detailed analysis indicates, that interlayer coupling constant $I^{\alpha,\beta}_{1,2}$ is strongly influenced by changes of the spectral dimension of the spacer layer. In the case of $\alpha = 3$, $\beta = 0$ the envelope function falls off with the spacer thickness d as $I_{1,2} \sim d^{-2}$, while for $\alpha = 2$ it decays as $I_{1,2} \sim d^{-1}$. Thus the strength of the interlayer coupling varies during dimensional crossover. This indicates a new way, in which properties of magnetic multilayers can be manipulated. In many layered systems the spectral dimension changes (dimensional crossover), when some external parameters like e.g. temperature or magnetic field are varied. Thus, by proper choice of the external fields, we are able to influence the strength of interlayer coupling, an effect important in the construction of electronic devices. The interface roughness ($\beta > 0$) acts in a similar way as decreasing α.

In conclusion, we have calculated the RKKY exchange integral in the case of a system that exhibits fractional dimensionality. Expression (14) for this integral has the same structure as the result obtained in the case of integral dimension [10], thus when α approches an integral value (2 or 3), our formulae reduce to the results obtained earlier. Having calculated the RKKY exchange integral in an αD system, by summation over all spin pairs we derive expression for the interlayer coupling in an αD superlattice. Both situations of fractal and non-fractal interfaces are considered. From the analysis of the formulae obtained it follows that dimensional crossover in a multilayer offers the possibility to control the strength of the interlayer interaction. Recent experimental data for the Eu/Se superlattice [23] confirm fractional dimensionality of this system.

Acknowledgements

Financial support from the State Committee for Scientific Research (KBN), through University (Z.B., R.J.) and KBN-Nr: 2 P03B 032 13 (W.G.) grants, is gratefully acknowledged. We thank Prof. H. Szymczak for pointing out to us Ref. [13].

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